

JOHANNIS WALLISII S.T.D.

In Celeberrima

ACADEMIA OXONIENSI

GEOMETRIÆ

Professoris Saviliani

EXERCITATIONES Tres.

De COMETARUM Distantiis Investigandis.

De RATIONUM & FRACTIONUM  
Reductione.

De PERIODO JULIANA.



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LONDINI,

Prostant Venales apud Mosem Pitt ad Insigne Angeli  
in Cœmeterio Divi Pauli, 1678.

20

TO THE HONORABLE

THE SENATE

OF THE UNIVERSITY OF OXFORD

PRESENTED

BY

JOHN B. B. B. B.

OF THE UNIVERSITY OF OXFORD

IN THE YEAR 1811

TO THE SENATE



FORWARDED

TO THE SENATE

JEREMIÆ HORROCCII,  
LIVERPOLIENSIS ANGLI, ex Palatinatu  
LANCASTRIÆ,  
OPERA POSTHUMA;  
VIZ.

Astronomia *Kepleriana*, Defensa & Promota.  
Excerpta ex Epistolis ad *Crabtræum* suum.  
Observationum Cœlestium Catalogus.  
Lunæ Theoria Nova.

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Accedunt

GUILIELMI CRABTRÆI, *Mancestriensis*,  
Observationes Cœlestes.

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*Quibus Accesserunt,*

JOHANNIS FLAMSTEDII, *Derbiensis*,  
De Temporis *Æquatione* Diatriba.  
Numeri ad Lunæ Theoriam *Horroccianam*.

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In Calce adjiciuntur, nondum Editæ,

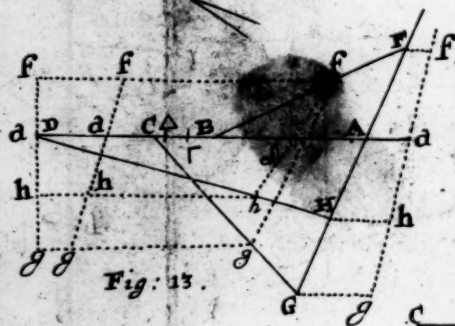
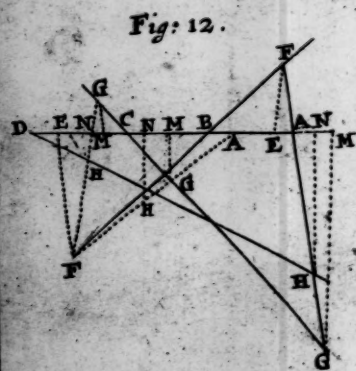
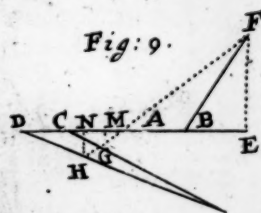
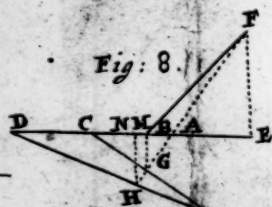
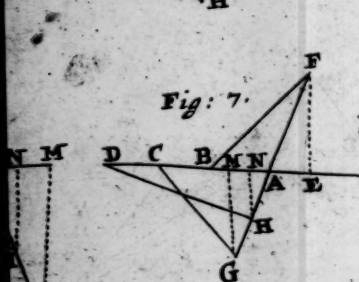
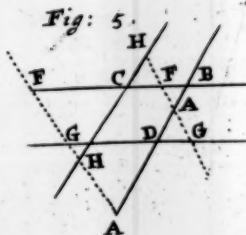
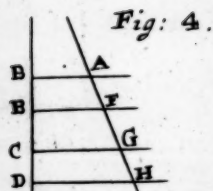
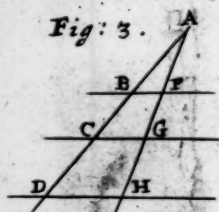
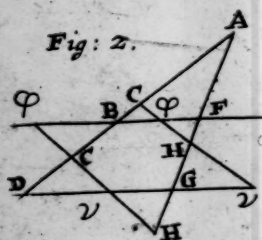
JOHANNIS WALLISII, S.T.D.

In Celeberrima Academia *Oxonienfi* GEOMETRIÆ  
Professoris *saviliane* Exercitationes Tres; viz.

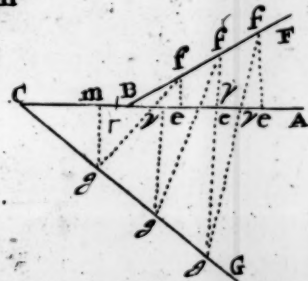
De *Cometarum* Distantiis Investigandis.  
De *Rationum* & *Fractionum* Reductione.  
De *Periodo Juliana*.

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LONDINI, Prostant venales apud *Mosem Pitt* ad Insigne  
*Angeli* in Cœmeterio D. *Pauli*, MDC LXX VIII.



*Figura: 15.*







D. JOHANNES WALLIS. S. T. D.

Geometriae Professor Savilianus, in Celeberrima  
Academia Oxoniensi;

Clarissimo Viro Johanni Collins, S.

Marii 25, 1677

**A**Nte Annos plus minus quindecim (si bene memini)  
aut etiam plures (Anno 1661, aut 1662,) Pro-  
blema (quod memoras) investigandum mihi propo-  
suit Vir Insignissimus D. Christophorus Wren, Astrono-  
miae (tum temporis) apud nos Professor Savilianus; ut quod  
usui foret, Cometarum à Terra distantis investigandis. Quod-  
que tum tulit ille responsum, jam libet (ex Adversariis delcri-  
ptum) Tibi etiam quærenti hic reponere.

circa

(ob y/a alia,  
infalsh mresh-  
gatem.)

Problema.

Expositis, in eodem Plano, quatuor Rectis positione datis (ut  
BA, BF, CG, DH, Fig. 1.) Quintam invenire (ut AFGH)  
quæ ab expositis ita secetur, ut interjecta segmenta sint in ratio-  
ne datà: (putà, AF, AG, AH, in ratione 1, m, n.)

FIG. I.

Solutio.

Intelligatur factum quod postulat. Et in expositarum al-  
teram, ut BA ductæ intelligantur perpendiculares FE, GM, HN.  
Reliquæque expositæ BF, CG, DH, ipsi BA occurrentes in B, C, D.

B o

Ponamus,

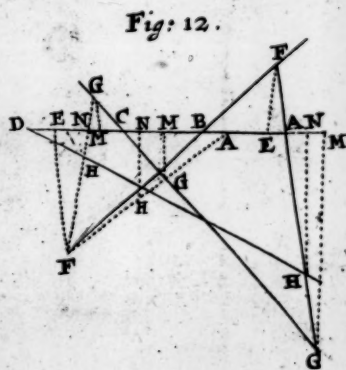
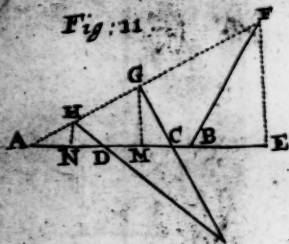
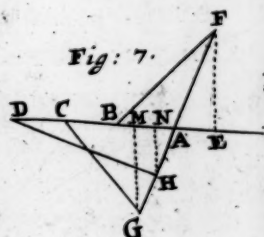
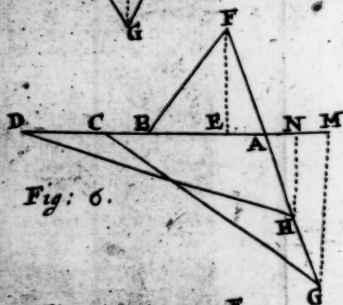
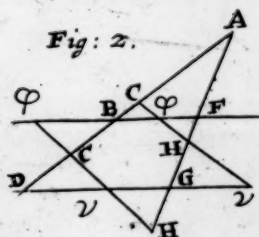
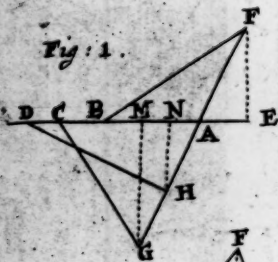


Fig: 3.

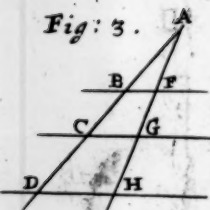


Fig: 4.

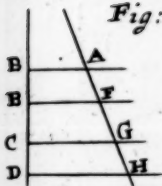


Fig: 5.



Fig: 8.



Fig: 9.

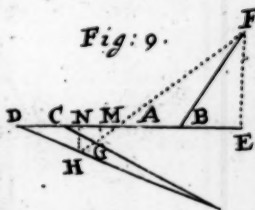


Fig: 10.



Fig: 14.



Fig: 13.

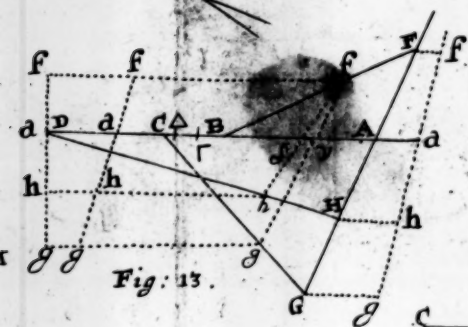
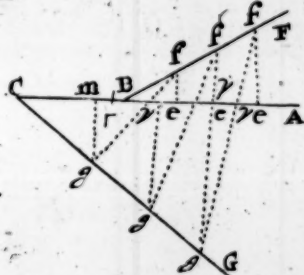


Figura: 15.



Ponamus,

$$FE = e. \quad AE = a. \quad BC = c. \quad BD = d. \quad \text{Et } \begin{cases} BE.EF :: f.t. \\ CM.MG :: g.i. \\ DN.NH :: h.i. \end{cases}$$

Ergo,

$$\begin{aligned} GM &= me. \quad AM = ma. \quad CE = c + fe. \quad DE = d + fe. \\ HN &= ne. \quad AN = na. \\ BE &= fe. \quad EM = a + ma. \quad CE - ME = CM. \quad DN = DE - EN. \\ CM &= gme. \quad EN = a + na. \quad c + fe - a - ma = gme. \quad hne = d + fe - a - na \\ DN &= hne. \quad c + fe - gme = a + ma. \quad a + na = d + fe - hne \\ \frac{c + fe - gme}{m + 1} &= a = \frac{d + fe - hne}{n + 1} \end{aligned}$$

Ponamus deinde (ad abbreviandum Calculum)  $m + 1 = k$ , &  $n + 1 = l$ .

Ergo,

$$\begin{aligned} lc + lfe - lgme &= kd + kfe - kbne \\ lc - kd &= kfe - lfe - kbne + lgme \\ \frac{lc - kd}{kfe - lfe - kbne + lgme} &= e. \end{aligned}$$

Habetur igitur quantitas  $e = FE$ : Adeoque Punctum  $F$ , & inde Puncta reliqua, ipsaque  $A, F, G, H$  quaesita.

Notandum interim, pro vario situ Punctorum  $C, D$ , ad  $B, A$ , variari signa  $+$   $-$ : Ita tamen ut Calculus nihilo inde evadurus sit perplexior; sed eodem planè modo processurus.

Puncta  $A, E, M, N$ , eundem inter se ordinem retinent, quem  $A, F, G, H$ ; qui ex datis statim constat. Pro vario autem illorum situ, Equationes conspiciuntur in hac varietate subscripta; nempe,

Ordo.

Equatio.

EAN

$$a + na =$$

ENA

$$a - na =$$

NEA

$$na - a =$$

EAM

$$a + ma =$$

EMA

$$a - ma =$$

MEA

$$ma - a =$$

CBE

$$c + fe =$$

BCE

$$c - fe =$$

BEC

$$fe - c =$$

DBE

$$\begin{array}{rcl}
 \text{DBE} & d + fe = & \\
 \text{BDE} & d - fe = & \\
 \text{BED} & fe - d = &
 \end{array}
 \left. \vphantom{\begin{array}{rcl} \text{DBE} \\ \text{BDE} \\ \text{BED} \end{array}} \right\} \text{DE.}$$

Exeipiendus autem est a Calculo supra posito casus unus aut alter: Nempe, ubi expositarum quatuor Rectarum, vel ternæ, vel bis binæ, sunt invicem Parallelæ. Quippe hoc casu, vel nulli expositarum occurrent tres reliquæ, vel non erit Problema in omni ratione expositâ possibile. Quoniam trium Parallelismus determinabit duorum segmentorum ad invicem rationem; & quidem omnium quatuor Parallelismus, determinabit omnium segmentorum inter se rationem. Sin habeantur bis binæ Parallelæ, nulla omnium tribus reliquis occurret.

## Scholium.

Si libent varios casus sigillatim expendere; sic esto.

Si expositarum duæ sint Parallelæ; putà BF, DG; (Fig. 2.) FIG. II. quas secet reliquæ DBA, CH: Facilius adhuc est operatio. Nam, propter datam rationem GF ad GA, (adeoque DB ad DA,) habetur inde Punctum A. Item, propter datam rationem FG ad FH, (adeoque CH ad CH,) habetur inde punctum H. Adeoque recta AFGH.

Si expositarum tres sint parallelæ; putà BF, CG, DH; (Fig. 3.) FIG. III. quas secet BA: ratio segmentorum FG, FH, non est pro arbitrio assignabilis, sed determinatur; quippe eadem erit atque BC, BD. Item, propter datam rationem aliam, putà GF ad GA, adeoque (quæ eadem est) CB ad CA; habetur punctum A. Sed sola recta AFGH, non determinatur. Quippe recta quævis ab A ducta (expositas secans) præstat quæsitum.

Si expositæ quatuor sint omnes parallelæ (ut Fig. 4.) non potest alia assignari ratio possibilis, præter eam quæ est distantiarum quas habent ipsæ inter se expositæ rectæ. Quippe quæcunque recta eas secet, segmenta interjecta habebit in ea ratione. FIG. IV.

Si expositarum quatuor, bis binæ sint parallelæ; putà BF, DG; & BA, CH, (Fig. 5.) Potest quidem ratio quævis assignari; FIG. V.

gnari; sed, in constructione, nulla omnium reliquas tres secabit. Verum, hujus loco, facilius succedet constructio. Nempe (ut modo ostensum est) propter datam rationem GF ad GA, (adeoque DB ad DA,) habetur punctum A. Item propter datam rationem AH ad AF, (adeoque BC ad BF,) habetur punctum F. Adeoque quaesita recta AF.

Quod autem attinet ad signa  $+$  — (in constructione generali) quæ pro vario situ Literarum mutanda erunt: nequid adhuc incertum maneat, hoc monemus.

FIG. VI.

1. Quantum ad quantitates EM & EN: Cum sit punctorum A, F, G, H, inter se ordo, ex datis segmentorum inter se rationibus notus; sitque idem ordo punctorum A, E, M, N; (ut per se patet:) etiam horum ordo pariter notus erit: Atque inde signorum  $+$  — ratio. Nempe,

FIG. VII.

Si sit FAG, FGA, GFA. Item FAH, FHA, HFA. Adeoque EAM, EMA, MEA. Et EAN, ENA, NEA, EMeft, EA  $+$  AM, EA — AM, MA — EA. EN est EA  $+$  AN, EA — AN, AN — EA. Hoc est  $a + ma$ ,

Fig. 6, 7, 8, 9, 12.  $a - ma$ ,  $ma - a$ ,  $a + na$ ,  $a - na$ ,  $na - a$ .

Fig. 10, 11, 12. Fig. 1, 2. Fig. 1, 6, 7, 9, 10. Fig. 11, 12. Fig. 1, 2.

Quodcunque autem horum contigerit, ponamus  $EM = a \pm ma = ka$ , &  $EN = a \pm na = la$ . Sive quod eodem recidit,  $1 \pm m = k$ , &  $1 \pm n = l$ .

FIG. VIII.

2. Quantum ad  $CE = fe \pm c$ , &  $DE = fe \pm d$ ; in quorum utroque triplex casus occurrit; puta,  $fe + c$ ,  $fe - c$ ,  $c - fe$ ; item  $fe + d$ ,  $fe - d$ ,  $d - fe$ : Cum incertum sit ex datis (donec Analyfi detegatur) utrum E contingit ultra B (puta dextrorsum) an ex parte contraria; adeoque utrum ponendum sit  $+ fe$ , an  $- fe$ : Ponamus, ad libitum, utrumvis; puta, ultra

FIG. IX.

B dextrorsum,  $BE = + fe$ : (Unde, si contingat contrarium esse verum, id indicabit quantitas  $fe$  tandem prodians negativa; adeoque situ suppositioni contrario sumenda.) Adeoque triplex hic casus, ad duplicem redigitur. Nempe,  $CE = BE \pm BC$

FIG. X.

$= fe \pm c$ . Hoc est,  $CE = BE + CE = fe + c$ , si contingat C sinistrorsum à B, non ad eas partes ad quas supponitur E: &  $BE - CE = fe - c$ , si supponantur E & C ad eadem partes à B:

&amp;



& quamquam fieri potest ut hoc casu C contingat, non modò ultra FIG. XI.  
B, sed & ultra E; hoc tamen Calculum non omnino turbat;  
quippe hoc solum inde consequitur, quod  $BE - BC = fe - c$   
siet revera quantitas negativa. (Et similiter  $DE = BE + BD =$   
 $fe + d$ ; (quod eodem plane modo intelligendum est, ut de CE  
jam dictum est.) Fieri autem potest, ut casus alter de C contin-  
gat, alter de D: putà, si C sit dextrorsum à B, & D sinistror-  
sum; (vel contra.) Quippe tum erit  $CE = BE - BC = fe - c$   
&  $DE = BE + BD = fe + d$ . Quæ omnia, pro situ puncto-  
rum B, C, D, ipso statim intuitu determinantur.

3. Quantum ad CM & DN: Id quidem incertum est (donec  
Analysi id constet) an futurum sit  $CM = CE + EM$ , an  $CE$   
 $- EM$ : (& similiter de DN.) Quoniam, licet ordo literarum  
A, E, M, N, inter se; determinatur ex ordine literarum A, F, G, H;  
tamen, an prorsum, an retrorsum, sumendus sit; non constat,  
priusquam constet an A sit ultra vel citra punctum E; (utrumvis  
enim contingere potest, pro variis rationibus assignatis, etiam  
eodem manente puncto F.) Constat autem utrum M & N sint  
ad easdem utraque, an ad contrarias partes. Licebit igitur, pro FIG. XII.  
arbitrio, ponere (verbi gratià)  $CM = CE + EM$  vel  $CE$   
 $- EM$ ; (& deinde DM, vel eodem, vel contrario signo, pro-  
ut fuerint tum CM, tum DN, vel utraque ad easdem, vel utra-  
que ad contrarias partes ipsius E; vel illa quidem ad contrarias,  
hæc ad easdem; vel contra.) Nihil aut hinc obyenet incommo-  
di: quippe id solum fiet (si contrario signo quam revera sit sup-  
ponetur,) quod, pro  $+a$ , proveniet utrobique  $-a$ . Semper  
utique hæc proveniet æquatio, 
$$\frac{ic + kd}{kf + lf - kb + gml} = e.$$

In qua æquatione, si Fractionis Numerator & Denominator fiat  
vel uterque affirmativus, vel uterque negativus, erit  $e$  quantitas  
affirmativa: sin fuerit eorum affirmativus alter, alter negativus,  
erit  $e$  quantitas negativa; & propterea situ suppositioni contra-  
rio sumenda: Putà, si F supponatur in BF supra BA, continget  
reverà ejus continuatione infra BA: & contra.

In Figuris 6, 7, 8, 9, 10, 11, 12 (cæteris manentibus) FA  
varium situm obtinet. Nempe, Fig. 6. cadit extra E; Fig. 7, 8.  
inter E & B; (illic post, hic ante, intersectiones rectarum CG,  
DH;) Fig. 9, inter B & C. Fig. 10. inter C & D. Fig. 11, ul-  
tra



tra D. Fig. 12. plures casus exhibentur. Reliquis Figuris, alii subinde casus exhibentur.

Intersecciones rectarum C G, D H, expositarum, antequam ad quæsitam quintam perveniant, indicantur ex alio ordine punctorum A, F, G, H, quam est punctorum A, B, C, D. Quod ipsa Schematum intuitu manifestum est.

*Idem Problema aliis constructum.*

Expositis, in eodem Plano (Fig. 16.) quatuor Rectis; B A, & (huic in B, C, D, occurrentes) B F, C G, D H; positione datis: Quæritur Quinta, ut F A G H, quæ expositas in F, A, G, H, ita secet ut Intersegmenta sint in ratione data; puta, A F, A G, A H, in ratione 1, m, n.

*Constructio.*

In rectâ infinitâ (quæ expositam A B utcunque secet; puta in a,) sumantur a f, a g, a h, in eadem ratione, & eodem ordine, cum ipsis A F, A G, A H, intersegmentis; quorum ratio & ordo supponitur dari. Unde ducantur f f, g g, h h, ipsi B A parallelae, expositis A F, A G, A H, occurrentes in f, g, h.

Quæ quidem puncta si fuerint in eadem rectâ, habetur quæsitum. Quippe hujus rectæ segmenta, erunt (propter parallelas) suis respectivè altitudinibus, hoc est, ipsarum parallelarum distantis, proportionales; adeoque in ratione data.

FIG. XIII.

Si verò non fuerint ipsa f, g, h, in una eademque rectâ, saltem erunt B f, C g, D h, ipsis B F, C G, D H, proportionales: quippe tum hæ, tum illæ, altitudines habebunt intersegmentis A F, A G, A H, proportionales. Quo casu,

Jungantur f g, f h, secantes ipsam B A in γ, δ; quæ propterea in eadem ratione (ab ipsa A B) sectæ erunt, quæ secantur F G & F H (respectivè) in A. Atque in eadem secentur (respectivè) B C, B D, in r, Δ. Puta, ut F A, ad A G, sic f γ ad r g, & B F ad r C: & ut F A ad A H, sic f δ ad Δ h, & B A ad Δ D. Deinde sumantur, ut Δ γ minus r γ, ad Δ r, sic B γ ad B F, & C γ ad C G, vel D H ad D H. Juncta F G, vel F H, erit ipsa F A G H quæsitâ. Quippe, hoc casu, ipsa γ, δ puncta (quæ alibi diversa sunt) coincident in eodem A puncto.

A F.

$$AF. AG. AH (:: i. m. n) :: af. ag. ah.$$

$$AF. AG :: af. ag :: \gamma f. \gamma g :: \Gamma B. \Gamma C.$$

$$AF. AH :: af. ah :: \delta f. \delta h :: \Delta B. \Delta D.$$

$$\Delta \delta - \Gamma \gamma. \Delta \Gamma :: Bf. BF :: Cg. CG :: D h. DH.$$

## Scholium.

Ratio hujus operationis, hæc est: Quoniam, sumpto ubivis, in  $BF$ , puncto  $F$ , sumi poterit (modo jam indicato) punctum  $g$  in  $CG$ , quod ducta recta  $fg$ , rectæ  $AB$  occurrens in  $f$ , segmenta habebit  $\gamma f, \gamma g$ , ipsis  $AF, AG$ , proportionalia. Item in  $DH$  sumetur punctum  $h$ , quod ducta  $fh$ , rectæ  $BA$  occurrens in  $\delta$ , segmenta faciet  $\delta f, \delta h$ , ipsis  $AF, AH$ , proportionalia. Id solum deest ad solvendum Problema, ut ipsa  $\delta \gamma$  puncta coincident: Quod cum in unico puncto contingat, illud sic porro investigabitur.

Nempe, vel sumpto alio quovis in  $BF$  puncto  $f$ , & huic respondentibus  $g, h$ , unde nova etiam emergent  $\gamma, \delta$ : Vel (quod eodem recidit) divisâ rectâ  $BC$  in  $\Gamma$ , sicut dividenda erat  $fg$  in  $\gamma$ ; &  $BD$  in  $\delta$ , sicut dividenda erat  $fh$  in  $\delta$ . (Quod tantundem est atque si intelligatur rectæ  $BF$  punctum  $f$  in ipso  $B$ ; quo casu intelligenda erit  $gh$ , in  $CD$ .) Cum igitur id queratur, ut punctis  $\Delta \Gamma$  vel  $\delta \gamma$ , coincidentibus evanescat distantia  $\Delta \Gamma$  seu  $\delta \gamma$ : Cumque fuerit, sumpto in (in  $BF$ ) puncto  $B$ , distantia illa  $\Delta \Gamma$ ; sumpto vero puncto  $f$ , distantia  $\delta \gamma$ , minor quam  $\Delta \Gamma$ , (donec, puncto  $\delta$  transgresso punctum  $\gamma$ , distantia iterum continuo crescat,) tanto scilicet quanto punctum  $\delta$  appropinquaverit puncto  $\gamma$ ; hoc est, quanto minor est  $\delta \gamma$  quam  $\Delta \Gamma$ , seu (quod potius dixerim)  $\Gamma \gamma$  quam  $\Delta \delta$ : Sitque hoc Decrementum. (quod mox probandum erit) proportionale ipsi  $BF$  rectæ; (seu, quod idem est, rectæ  $Cg$ , vel  $Dh$ , quippe has invicem Proportionales esse, ex jam dictis satis constat.) Erit, ut  $\Delta \delta - \Gamma \gamma$ , ad  $\Delta \Gamma$ ; sic  $Bf$  ad  $BF$ . Hoc est, si distantia  $\Delta \Gamma$ , propter rectam  $BF$ , minuatur quantum est  $\Delta \delta - \Gamma \gamma$ ; evanescet tota  $\Delta \Gamma$  (coincidentibus  $\Delta \Gamma$  punctis) posito  $f$  in  $F$ ; sumpta scilicet  $BF$  ad  $BF$ , in ea ratione qua est  $\Delta \Gamma$  ad  $\Delta \Gamma - \delta \gamma$ , seu potius  $\Delta \delta - \Gamma \gamma$ .

:: (Dico autem  $\Delta \delta - \Gamma \gamma$ , potius quam  $\Delta \Gamma - \delta \gamma$ , ut perinde sit sive ita sumatur  $f$ , ut  $\delta$  transiverit  $\gamma$ , necne. Quanquam enim inde mutetur situs puncti  $F$ ; nempe, si transiverit, erit  $F$  citra  $f$ ; sin minus, ultra: Proportio tamen  $Bf$  ad  $BF$  utrobique manet

quam

si dicamus  $\Delta\delta - \Gamma\gamma$ ; non autem si dicatur  $\Delta\Gamma - \delta\gamma$ . Si enim  $\Delta\delta - \Gamma\gamma$  minor sit,  $\Delta\Gamma$ , erit *f* citra *F*; si major, ultra; si æqualis, coincident.)

Si vero contingat, contra quam supponitur,  $\Gamma\gamma$  majorem esse quam  $\Delta\delta$ , (adeoque  $\Delta\delta - \Gamma\gamma$  quantitatem esse negativam, seu minorem quam nihil;) Indicio hoc erit, non sumendum esse punctum *F* ad eas à *B* partes prout erat suppositum, (putà sursum versus *f*;) sed ad partes contrarias; putà in *fB* deorsum continuata.

a ia

c FIG. XIV.

n  
n  
figu  
n  
Δ

Quod vero suscepimus demonstrandum; nempe, quod Decrementa  $\Delta\delta - \Gamma\gamma$  sint ipsis *Bf* rectis proportionalia, ostenditur. Sumantur in *BF* (Fig. 14.) quotlibet puncta *f*; & in *DH*, totidem quæ illis correfpondeant puncta *h*. Erunt (propter similes sectiones rectarum *fb*, adeoque altitudines proportionales) rectæ *Dh* ipsis *Bf* proportionales. Sed & (propter similem ubique sectionem rectarum *BD* in  $\Delta$ , & *fb* adeoque *en* in  $\delta$ ,) erunt ubique *en*,  $\delta\delta$  ipsis *BD*,  $B\Delta$  proportionales. Hoc est  $DB : Be - Dn (= en),$  &  $\Delta B + Be - \Delta\delta (= \delta\delta),$  ipsis *BD*,  $B\Delta$  proportionales. Ergo (dividendo)  $Be - Dn$  &  $Be - \Delta\delta$ , sive (mutatis figuris)  $Dn - Be$  &  $\Delta\delta - Be$ , iidem *BD*,  $B\Delta$  proportionales. Sunt autem (ut modo demonstratum est) tum *Dn*, tum *Be*, adeoque  $Dn - Be$ , ipsis *Bf* proportionales: Ergo &  $\Delta\delta - Be$ ; adeoque &  $\Delta\delta$  (quia *Be*) sunt ipsis *Bf* proportionales.

Δ

δ δ Δ

FIG. XV.

Eodem modo ostendetur, rectas quotlibet  $\Gamma\gamma$  ipsis *Bf* respectivè sumptis proportionales esse.

$FG : FA : AG :: fg : fa : ag :: em : ey : ym :: BC : Br : rC :: BC : Br :: em (= CB + Be - Cm) : rC (= rB + Be - rY) :: Cm - Be : rY - Be.$

$Bf : Bf :: Be : Be :: Cg : Cg :: Cm : Cm :: Cm - Be : Cm - Be :: \Gamma\gamma - Be : rY - Be :: rY : rY.$

e

Ergo

Ergo (proportionalium differentia)  $\Delta\delta - \Gamma\gamma$ , iisdem Bf proportionales erunt. Quod demonstrandum suscepimus.

Bf. Bf. :: ( $\Delta\delta . \Delta\delta :: \Gamma\gamma . \Gamma\gamma ::$ )  $\Delta\delta - \Gamma\gamma . \Delta\delta - \Gamma\gamma$ .

Pro vario situ punctorum B, C, D; item pro vario ordine punctorum A, F, G, H; oriuntur variae signorum — — mutationes: sed idem manebit Demonstrationis Processus eodem planè modo quo in priori methodo ostensum est.

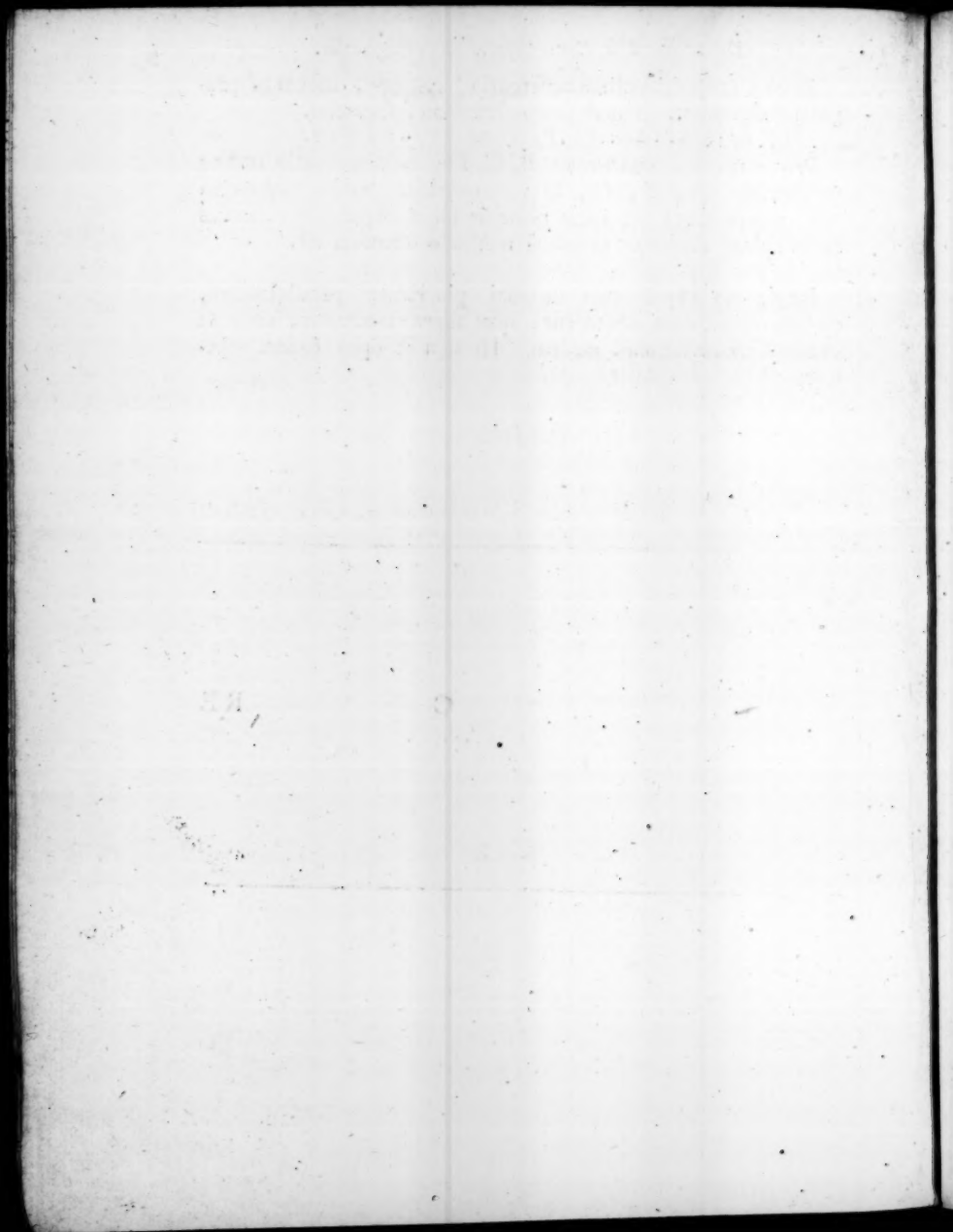
Item, ob expositarum duarum pluriùve parallelismum, eadem mutationes obvenient, quæ supra traduntur; unde & methodi abbreviationes eadem. Ut non sit opus eadem repetere.

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C

RE

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# REDUCTIO

*FRACTIONUM* seu *RATIONUM*

AD

**Proximè-æquales in minimis Terminis.**

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REDUCTION

THE REDUCTION OF THE

AND

THE REDUCTION OF THE

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C 2





JOHANNES COLLINS, Lectori, S.

**I**N *Astronomia non raro occurrit Usus Rationum seu Proportionum Partium exprimendarum Systematis Planetarii ad semetipsas invicem, vel etiam unius Systematis ad alterum, quin autem Rationum Distantiarum & Magnitudinum diversorum Corporum Cœlestium. Hoc ut peragatur minimis Terminis, ac ut Fractiones, sive Numeri evitentur majores, non delectabile tantum verum & perquam utile audit Opus. Præ manibus igitur sequentem habens Doctoris Wallisii Viri undiquaque Doctissimi Diatribam, nihilq; unquam, quod extat, ejusdem naturæ observans, hoc dignum Typis mandetur existimavi: Nequaquam ambigens quin perplacuit Astronomo, Mœtiii observare Numeros de Proportionem Diametri Circuli ad Circumferentiam derivatos fuisse (quod & jam fit, & de quamplurimis aliis fiat numeris) ab istis Ludolphi Van Ceulen Notissimis.*

*Reductio*





## Reductio Fractionum seu Rationum ad proximè- equales in Minoribus Terminis.

### Problema.

**E**Xposita Fractione quavis (puta  $\frac{444444}{376171}$ ;) Fractionem invenire, quæ sit vel expositæ æqualis, si fieri possit, vel saltem quæ expositam vel proximè superet, vel ab ea proximè deficiat, Denominatorem habens dato numero non majorem: (puta, qui numerum 999 non superet, seu tres locos non excedat:) sitque in terminis minimis.

Præmitto Lemmatjs loco, tanquam satis notum, vel facile (si sit opus) demonstratu.

*Si Fractionis duo Termini (Numerator & Denominator) æqualiter utrunque multiplicentur; idem, qui prius, manet Fractionis valor: Si verò inæqualiter, valor variatur: Et quidem augetur, si majore Multiplicatore multiplicetur Numerator quam Denominator: Minuitur, si contra.*

Seu (quod eodem recidit;)

*Si Fractionis duo Termini (Numerator & Denominator) ita auquantur ambo, ut Augmenta sint ipsis Terminis proportionalia; idem, qui prius, manet Fractionis valor: Augetur vero valor, si Numeratoris Augmentum, ad Augmentum Denominatoris, majorem rationem habeat, quam habet ipse Numerator ad Denominatorem; Minuitur, si minorem.*

Putat

$$\text{Puta } \frac{1}{3} = \frac{1 \times 2}{3 \times 2} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} \&c.$$

$$\text{Seu } \frac{1}{3} = \frac{1+1}{3+3} = \frac{1+2}{3+6} = \frac{1+3}{3+9} \&c.$$

$$\text{Sed } \frac{1}{3} < \frac{1 \times 3}{3 \times 2} = \frac{3}{6} = \frac{1+2}{3+3}.$$

$$\text{Et } \frac{1}{3} > \frac{1 \times 2}{3 \times 3} = \frac{2}{9} = \frac{1+1}{3+6}.$$

$$\frac{n}{d} = \frac{2n}{2d} = \frac{3n}{3d} = \frac{an}{ad}$$

$$\frac{n}{d} = \frac{n+n}{d+d} = \frac{n+2n}{d+2d} = \frac{n+an}{d+ad}$$

$$\frac{n}{d} < \frac{3n}{2d} = \frac{n+2n}{d+d}$$

$$\frac{n}{d} > \frac{2n}{3d} = \frac{n+n}{d+2d}$$

His positis; Si exposita Fractio, ad minimos terminos reducta, (utroque scilicet termino per maximum cōm̄munem Divisorem diviso,) Denominatorem habeat imperato non majorem. Habetur quæsitum. Est utique (ut patet) Fractio sic reducta, expositæ æqualis, & qualis quæritur.

Si verò exposita Fractio, ad minimos terminos reducta, Denominatorem habeat imperato majorem: Quærenda est, quæ vel *proxime major* sit, vel *proxime minor*, Denominatorem habens qualis imperatur.

*Fractionem proxime majorem, sic inquirō.*

Denominatorem Fractionis expositæ (illiusve ad quam reducitur, si reductio prius instituta fuerit) per ipsius Numeratorem divido; ut habeam, qui Numeratori respondeat, Denominatorem, in Integris cum partibus Decimalibus annexis, satis accuratum.

Putā, 2684769) 8376571 (3, 12003416 +.

Nam, ut 684769, ad 8376571; sic 1, ad 3, 12003416 +.

Adeoque

Adeoque, pro exposita Fractione  $\frac{2444762}{8376371}$ , substituo (huic satis accuratè æqualem)  $\frac{1}{3,12003416}$ , Numeratorem habentem 1. Quam Fractionem Primam Completam appello: eandemq; Denominatoris appendice partium Decimalium truncatam,  $\frac{1}{3}$ , appello Fractionem Primam Truncatam; ejusque partes Decimales amputatas, 0,12003416, Appendicem appello, seu Mantissam Primam.

Estque hæc Truncata Fractio,  $\frac{1}{3}$ , tum justo major (propter Denominatorem justo minorem, utpote partibus Decimalibus multatum,) tum proximè major, habentium Numeratorem 1, & Denominatorem numerum integrum: (quippe  $\frac{1}{3}$  adhuc major est; &  $\frac{1}{4}$  justo minor; & de reliquis similiter:) Neque ex justo majoribus, propius ad veritatem accedere potest ulla, nisi aucto Numeratore 1.

Deinde loco Numeratoris 1, sumptis Numeratoribus, 2, 3, 4, &c. Fractiones

his convenientes erunt  $\frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \frac{5}{15}, \frac{6}{18}, \frac{7}{21}, \frac{8}{24}, \frac{9}{27}, \frac{10}{30}, \frac{11}{33}, \frac{12}{36}, \frac{13}{39}, \frac{14}{42}, \frac{15}{45}, \frac{16}{48}, \frac{17}{51}, \frac{18}{54}, \frac{19}{57}, \frac{20}{60}, \frac{21}{63}, \frac{22}{66}, \frac{23}{69}, \frac{24}{72}, \frac{25}{75}, \frac{26}{78}, \frac{27}{81}, \frac{28}{84}, \frac{29}{87}, \frac{30}{90}, \frac{31}{93}, \frac{32}{96}, \frac{33}{99}, \frac{34}{102}, \frac{35}{105}, \frac{36}{108}, \frac{37}{111}, \frac{38}{114}, \frac{39}{117}, \frac{40}{120}, \frac{41}{123}, \frac{42}{126}, \frac{43}{129}, \frac{44}{132}, \frac{45}{135}, \frac{46}{138}, \frac{47}{141}, \frac{48}{144}, \frac{49}{147}, \frac{50}{150}, \frac{51}{153}, \frac{52}{156}, \frac{53}{159}, \frac{54}{162}, \frac{55}{165}, \frac{56}{168}, \frac{57}{171}, \frac{58}{174}, \frac{59}{177}, \frac{60}{180}, \frac{61}{183}, \frac{62}{186}, \frac{63}{189}, \frac{64}{192}, \frac{65}{195}, \frac{66}{198}, \frac{67}{201}, \frac{68}{204}, \frac{69}{207}, \frac{70}{210}, \frac{71}{213}, \frac{72}{216}, \frac{73}{219}, \frac{74}{222}, \frac{75}{225}, \frac{76}{228}, \frac{77}{231}, \frac{78}{234}, \frac{79}{237}, \frac{80}{240}, \frac{81}{243}, \frac{82}{246}, \frac{83}{249}, \frac{84}{252}, \frac{85}{255}, \frac{86}{258}, \frac{87}{261}, \frac{88}{264}, \frac{89}{267}, \frac{90}{270}, \frac{91}{273}, \frac{92}{276}, \frac{93}{279}, \frac{94}{282}, \frac{95}{285}, \frac{96}{288}, \frac{97}{291}, \frac{98}{294}, \frac{99}{297}, \frac{100}{300}, \frac{101}{303}, \frac{102}{306}, \frac{103}{309}, \frac{104}{312}, \frac{105}{315}, \frac{106}{318}, \frac{107}{321}, \frac{108}{324}, \frac{109}{327}, \frac{110}{330}, \frac{111}{333}, \frac{112}{336}, \frac{113}{339}, \frac{114}{342}, \frac{115}{345}, \frac{116}{348}, \frac{117}{351}, \frac{118}{354}, \frac{119}{357}, \frac{120}{360}, \frac{121}{363}, \frac{122}{366}, \frac{123}{369}, \frac{124}{372}, \frac{125}{375}, \frac{126}{378}, \frac{127}{381}, \frac{128}{384}, \frac{129}{387}, \frac{130}{390}, \frac{131}{393}, \frac{132}{396}, \frac{133}{399}, \frac{134}{402}, \frac{135}{405}, \frac{136}{408}, \frac{137}{411}, \frac{138}{414}, \frac{139}{417}, \frac{140}{420}, \frac{141}{423}, \frac{142}{426}, \frac{143}{429}, \frac{144}{432}, \frac{145}{435}, \frac{146}{438}, \frac{147}{441}, \frac{148}{444}, \frac{149}{447}, \frac{150}{450}, \frac{151}{453}, \frac{152}{456}, \frac{153}{459}, \frac{154}{462}, \frac{155}{465}, \frac{156}{468}, \frac{157}{471}, \frac{158}{474}, \frac{159}{477}, \frac{160}{480}, \frac{161}{483}, \frac{162}{486}, \frac{163}{489}, \frac{164}{492}, \frac{165}{495}, \frac{166}{498}, \frac{167}{501}, \frac{168}{504}, \frac{169}{507}, \frac{170}{510}, \frac{171}{513}, \frac{172}{516}, \frac{173}{519}, \frac{174}{522}, \frac{175}{525}, \frac{176}{528}, \frac{177}{531}, \frac{178}{534}, \frac{179}{537}, \frac{180}{540}, \frac{181}{543}, \frac{182}{546}, \frac{183}{549}, \frac{184}{552}, \frac{185}{555}, \frac{186}{558}, \frac{187}{561}, \frac{188}{564}, \frac{189}{567}, \frac{190}{570}, \frac{191}{573}, \frac{192}{576}, \frac{193}{579}, \frac{194}{582}, \frac{195}{585}, \frac{196}{588}, \frac{197}{591}, \frac{198}{594}, \frac{199}{597}, \frac{200}{600}, \frac{201}{603}, \frac{202}{606}, \frac{203}{609}, \frac{204}{612}, \frac{205}{615}, \frac{206}{618}, \frac{207}{621}, \frac{208}{624}, \frac{209}{627}, \frac{210}{630}, \frac{211}{633}, \frac{212}{636}, \frac{213}{639}, \frac{214}{642}, \frac{215}{645}, \frac{216}{648}, \frac{217}{651}, \frac{218}{654}, \frac{219}{657}, \frac{220}{660}, \frac{221}{663}, \frac{222}{666}, \frac{223}{669}, \frac{224}{672}, \frac{225}{675}, \frac{226}{678}, \frac{227}{681}, \frac{228}{684}, \frac{229}{687}, \frac{230}{690}, \frac{231}{693}, \frac{232}{696}, \frac{233}{699}, \frac{234}{702}, \frac{235}{705}, \frac{236}{708}, \frac{237}{711}, \frac{238}{714}, \frac{239}{717}, \frac{240}{720}, \frac{241}{723}, \frac{242}{726}, \frac{243}{729}, \frac{244}{732}, \frac{245}{735}, \frac{246}{738}, \frac{247}{741}, \frac{248}{744}, \frac{249}{747}, \frac{250}{750}, \frac{251}{753}, \frac{252}{756}, \frac{253}{759}, \frac{254}{762}, \frac{255}{765}, \frac{256}{768}, \frac{257}{771}, \frac{258}{774}, \frac{259}{777}, \frac{260}{780}, \frac{261}{783}, \frac{262}{786}, \frac{263}{789}, \frac{264}{792}, \frac{265}{795}, \frac{266}{798}, \frac{267}{801}, \frac{268}{804}, \frac{269}{807}, \frac{270}{810}, \frac{271}{813}, \frac{272}{816}, \frac{273}{819}, \frac{274}{822}, \frac{275}{825}, \frac{276}{828}, \frac{277}{831}, \frac{278}{834}, \frac{279}{837}, \frac{280}{840}, \frac{281}{843}, \frac{282}{846}, \frac{283}{849}, \frac{284}{852}, \frac{285}{855}, \frac{286}{858}, \frac{287}{861}, \frac{288}{864}, \frac{289}{867}, \frac{290}{870}, \frac{291}{873}, \frac{292}{876}, \frac{293}{879}, \frac{294}{882}, \frac{295}{885}, \frac{296}{888}, \frac{297}{891}, \frac{298}{894}, \frac{299}{897}, \frac{300}{900}, \frac{301}{903}, \frac{302}{906}, \frac{303}{909}, \frac{304}{912}, \frac{305}{915}, \frac{306}{918}, \frac{307}{921}, \frac{308}{924}, \frac{309}{927}, \frac{310}{930}, \frac{311}{933}, \frac{312}{936}, \frac{313}{939}, \frac{314}{942}, \frac{315}{945}, \frac{316}{948}, \frac{317}{951}, \frac{318}{954}, \frac{319}{957}, \frac{320}{960}, \frac{321}{963}, \frac{322}{966}, \frac{323}{969}, \frac{324}{972}, \frac{325}{975}, \frac{326}{978}, \frac{327}{981}, \frac{328}{984}, \frac{329}{987}, \frac{330}{990}, \frac{331}{993}, \frac{332}{996}, \frac{333}{999}, \frac{334}{1002}, \frac{335}{1005}, \frac{336}{1008}, \frac{337}{1011}, \frac{338}{1014}, \frac{339}{1017}, \frac{340}{1020}, \frac{341}{1023}, \frac{342}{1026}, \frac{343}{1029}, \frac{344}{1032}, \frac{345}{1035}, \frac{346}{1038}, \frac{347}{1041}, \frac{348}{1044}, \frac{349}{1047}, \frac{350}{1050}, \frac{351}{1053}, \frac{352}{1056}, \frac{353}{1059}, \frac{354}{1062}, \frac{355}{1065}, 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\frac{576}{1728}, \frac{577}{1731}, \frac{578}{1734}, \frac{579}{1737}, \frac{580}{1740}, \frac{581}{1743}, \frac{582}{1746}, \frac{583}{1749}, \frac{584}{1752}, \frac{585}{1755}, \frac{586}{1758}, \frac{587}{1761}, \frac{588}{1764}, \frac{589}{1767}, \frac{590}{1770}, \frac{591}{1773}, \frac{592}{1776}, \frac{593}{1779}, \frac{594}{1782}, \frac{595}{1785}, \frac{596}{1788}, \frac{597}{1791}, \frac{598}{1794}, \frac{599}{1797}, \frac{600}{1800}, \frac{601}{1803}, \frac{602}{1806}, \frac{603}{1809}, \frac{604}{1812}, \frac{605}{1815}, \frac{606}{1818}, \frac{607}{1821}, \frac{608}{1824}, \frac{609}{1827}, \frac{610}{1830}, \frac{611}{1833}, \frac{612}{1836}, \frac{613}{1839}, \frac{614}{1842}, \frac{615}{1845}, \frac{616}{1848}, \frac{617}{1851}, \frac{618}{1854}, \frac{619}{1857}, \frac{620}{1860}, \frac{621}{1863}, \frac{622}{1866}, \frac{623}{1869}, \frac{624}{1872}, \frac{625}{1875}, \frac{626}{1878}, \frac{627}{1881}, \frac{628}{1884}, \frac{629}{1887}, \frac{630}{1890}, \frac{631}{1893}, \frac{632}{1896}, \frac{633}{1899}, \frac{634}{1902}, \frac{635}{1905}, \frac{636}{1908}, \frac{637}{1911}, \frac{638}{1914}, \frac{639}{1917}, \frac{640}{1920}, \frac{641}{1923}, \frac{642}{1926}, \frac{643}{1929}, \frac{644}{1932}, \frac{645}{1935}, \frac{646}{1938}, \frac{647}{1941}, \frac{648}{1944}, \frac{649}{1947}, \frac{650}{1950}, \frac{651}{1953}, \frac{652}{1956}, \frac{653}{1959}, \frac{654}{1962}, \frac{655}{1965}, \frac{656}{1968}, \frac{657}{1971}, \frac{658}{1974}, \frac{659}{1977}, \frac{660}{1980}, \frac{661}{1983}, \frac{662}{1986}, \frac{663}{1989}, \frac{664}{1992}, \frac{665}{1995}, \frac{666}{1998}, \frac{667}{2001}, \frac{668}{2004}, \frac{669}{2007}, \frac{670}{2010}, \frac{671}{2013}, \frac{672}{2016}, \frac{673}{2019}, \frac{674}{2022}, \frac{675}{2025}, \frac{676}{2028}, \frac{677}{2031}, \frac{678}{2034}, \frac{679}{2037}, \frac{680}{2040}, \frac{681}{2043}, \frac{682}{2046}, \frac{683}{2049}, \frac{684}{2052}, \frac{685}{2055}, \frac{686}{2058}, \frac{687}{2061}, \frac{688}{2064}, \frac{689}{2067}, \frac{690}{2070}, \frac{691}{2073}, \frac{692}{2076}, \frac{693}{2079}, \frac{694}{2082}, \frac{695}{2085}, \frac{696}{2088}, \frac{697}{2091}, \frac{698}{2094}, \frac{699}{2097}, \frac{700}{2100}, \frac{701}{2103}, \frac{702}{2106}, \frac{703}{2109}, \frac{704}{2112}, \frac{705}{2115}, \frac{706}{2118}, \frac{707}{2121}, \frac{708}{2124}, \frac{709}{2127}, \frac{710}{2130}, \frac{711}{2133}, \frac{712}{2136}, \frac{713}{2139}, \frac{714}{2142}, \frac{715}{2145}, \frac{716}{2148}, \frac{717}{2151}, \frac{718}{2154}, \frac{719}{2157}, \frac{720}{2160}, \frac{721}{2163}, \frac{722}{2166}, \frac{723}{2169}, \frac{724}{2172}, \frac{725}{2175}, \frac{726}{2178}, \frac{727}{2181}, \frac{728}{2184}, \frac{729}{2187}, \frac{730}{2190}, \frac{731}{2193}, \frac{732}{2196}, \frac{733}{2199}, \frac{734}{2202}, \frac{735}{2205}, \frac{736}{2208}, \frac{737}{2211}, \frac{738}{2214}, \frac{739}{2217}, \frac{740}{2220}, \frac{741}{2223}, \frac{742}{2226}, \frac{743}{2229}, \frac{744}{2232}, \frac{745}{2235}, \frac{746}{2238}, \frac{747}{2241}, \frac{748}{2244}, \frac{749}{2247}, \frac{750}{2250}, \frac{751}{2253}, \frac{752}{2256}, \frac{753}{2259}, \frac{754}{2262}, \frac{755}{2265}, \frac{756}{2268}, \frac{757}{2271}, \frac{758}{2274}, \frac{759}{2277}, \frac{760}{2280}, \frac{761}{2283}, \frac{762}{2286}, \frac{763}{2289}, \frac{764}{2292}, \frac{765}{2295}, \frac{766}{2298}, \frac{767}{2301}, \frac{768}{2304}, \frac{769}{2307}, \frac{770}{2310}, \frac{771}{2313}, \frac{772}{2316}, \frac{773}{2319}, \frac{774}{2322}, \frac{775}{2325}, \frac{776}{2328}, \frac{777}{2331}, \frac{778}{2334}, \frac{779}{2337}, \frac{780}{2340}, \frac{781}{2343}, \frac{782}{2346}, \frac{783}{2349}, \frac{784}{2352}, \frac{785}{2355}, \frac{786}{2358}, \frac{787}{2361}, \frac{788}{2364}, \frac{789}{2367}, \frac{790}{2370}, \frac{791}{2373}, \frac{792}{2376}, \frac{793}{2379}, \frac{794}{2382}, \frac{795}{2385}, \frac{796}{2388}, \frac{797}{2391}, \frac{798}{2394}, \frac{799}{2397}, \frac{800}{2400}, \frac{801}{2403}, \frac{802}{2406}, \frac{803}{2409}, \frac{804}{2412}, \frac{805}{2415}, \frac{806}{2418}, \frac{807}{2421}, \frac{808}{2424}, \frac{809}{2427}, \frac{810}{2430}, \frac{811}{2433}, \frac{812}{2436}, \frac{813}{2439}, \frac{814}{2442}, \frac{815}{2445}, \frac{816}{2448}, \frac{817}{2451}, \frac{818}{2454}, \frac{819}{2457}, \frac{820}{2460}, \frac{821}{2463}, \frac{822}{2466}, \frac{823}{2469}, \frac{824}{2472}, \frac{825}{2475}, \frac{826}{2478}, \frac{827}{2481}, \frac{828}{2484}, \frac{829}{2487}, \frac{830}{2490}, \frac{831}{2493}, \frac{832}{2496}, \frac{833}{2499}, \frac{834}{2502}, \frac{835}{2505}, \frac{836}{2508}, \frac{837}{2511}, \frac{838}{2514}, \frac{839}{2517}, \frac{840}{2520}, \frac{841}{2523}, \frac{842}{2526}, \frac{843}{2529}, \frac{844}{2532}, \frac{845}{2535}, \frac{846}{2538}, \frac{847}{2541}, \frac{848}{2544}, \frac{849}{2547}, \frac{850}{2550}, \frac{851}{2553}, \frac{852}{2556}, \frac{853}{2559}, \frac{854}{2562}, \frac{855}{2565}, \frac{856}{2568}, \frac{857}{2571}, \frac{858}{2574}, \frac{859}{2577}, \frac{860}{2580}, \frac{861}{2583}, \frac{862}{2586}, \frac{863}{2589}, \frac{864}{2592}, \frac{865}{2595}, \frac{866}{2598}, \frac{867}{2601}, \frac{868}{2604}, \frac{869}{2607}, \frac{870}{2610}, \frac{871}{2613}, \frac{872}{2616}, \frac{873}{2619}, \frac{874}{2622}, \frac{875}{2625}, \frac{876}{2628}, \frac{877}{2631}, \frac{878}{2634}, \frac{879}{2637}, \frac{880}{2640}, \frac{881}{2643}, \frac{882}{2646}, \frac{883}{2649}, \frac{884}{2652}, \frac{885}{2655}, \frac{886}{2658}, \frac{887}{2661}, \frac{888}{2664}, \frac{889}{2667}, \frac{890}{2670}, \frac{891}{2673}, \frac{892}{2676}, \frac{893}{2679}, \frac{894}{2682}, \frac{895}{2685}, \frac{896}{2688}, \frac{897}{2691}, \frac{898}{2694}, \frac{899}{2697}, \frac{900}{2700}, \frac{901}{2703}, \frac{902}{2706}, \frac{903}{2709}, \frac{904}{2712}, \frac{905}{2715}, \frac{906}{2718}, \frac{907}{2721}, \frac{908}{2724}, \frac{909}{2727}, \frac{910}{2730}, \frac{911}{2733}, \frac{912}{2736}, \frac{913}{2739}, \frac{914}{2742}, \frac{915}{2745}, \frac{916}{2748}, \frac{917}{2751}, \frac{918}{2754}, \frac{919}{2757}, \frac{920}{2760}, \frac{921}{2763}, \frac{922}{2766}, \frac{923}{2769}, \frac{924}{2772}, \frac{925}{2775}, \frac{926}{2778}, \frac{927}{2781}, \frac{928}{2784}, \frac{929}{2787}, \frac{930}{2790}, \frac{931}{2793}, \frac{932}{2796}, \frac{933}{2799}, \frac{934}{2802}, \frac{935}{2805}, \frac{936}{2808}, \frac{937}{2811}, \frac{938}{2814}, \frac{939}{2817}, \frac{940}{2820}, \frac{941}{2823}, \frac{942}{2826}, \frac{943}{2829}, \frac{944}{2832}, \frac{945}{2835}, \frac{946}{2838}, \frac{947}{2841}, \frac{948}{2844}, \frac{949}{2847}, \frac{950}{2850}, \frac{951}{2853}, \frac{952}{2856}, \frac{953}{2859}, \frac{954}{2862}, \frac{955}{2865}, \frac{956}{2868}, \frac{957}{2871}, \frac{958}{2874}, \frac{959}{2877}, \frac{960}{2880}, \frac{961}{2883}, \frac{962}{2886}, \frac{963}{2889}, \frac{964}{2892}, \frac{965}{2895}, \frac{966}{2898}, \frac{967}{2901}, \frac{968}{2904}, \frac{969}{2907}, \frac{970}{2910}, \frac{971}{2913}, \frac{972}{2916}, \frac{973}{2919}, \frac{974}{2922}, \frac{975}{2925}, \frac{976}{2928}, \frac{977}{2931}, \frac{978}{2934}, \frac{979}{2937}, \frac{980}{2940}, \frac{981}{2943}, \frac{982}{2946}, \frac{983}{2949}, \frac{984}{2952}, \frac{985}{2955}, \frac{986}{2958}, \frac{987}{2961}, \frac{988}{2964}, \frac{989}{2967}, \frac{990}{2970}, \frac{991}{2973}, \frac{992}{2976}, \frac{993}{2979}, \frac{994}{2982}, \frac{995}{2985}, \frac{996}{2988}, \frac{997}{2991}, \frac{998}{2994}, \frac{999}{2997}, \frac{1000}{3000}, \frac{1001}{3003}, \frac{1002}{3006}, \frac{1003}{3009}, \frac{1004}{3012}, \frac{1005}{3015}, \frac{1006}{3018}, \frac{1007}{3021}, \frac{1008}{3024}, \frac{1009}{3027}, \frac{1010}{3030}, \frac{1011}{3033}, \frac{1012}{3036}, \frac{1013}{3039}, \frac{1014}{3042}, \frac{1015}{3045}, \frac{1016}{3048}, \frac{1017}{3051}, \frac{1018}{3054}, \frac{1019}{3057}, \frac{1020}{3060}, \frac{1021}{3063}, \frac{1022}{3066}, \frac{1023}{3069}, \frac{1024}{307$

Quærendum igitur (quod dividendo investigabitur) quota Multiplicatione Denominatoris primi  $3,12003416$ , aliquid ex partibus Decimalibus transferatur ad Numeros integros. Diviſoque  $1$ , ſeu  $1,00000000$ , per  $0,12003416$ ; habetur Quotiens  $8,331$  —.

$0,12003416$ )  $1.(8,33096$  +.

Liquet igitur (hoc caſu) omnimum eſſe Numerum integrum, qui Denominatorem primum  $3,12003416$ , multiplicans, aliquid ex partibus Decimalibus transferet ad ſedem Integrorum. Et quidem vel Mantiffa  $0,12003416$

Noncuplum, vel ſaltem ipſius octuplum cum additamento ſufficiente, habendum eſſe, quo ex partibus Decimalibus tranſferatur 1 ad Integros. Eſt

$0,12003416$   
8.

$0,96027328$

$0,03972672$

$1,00000000$

utique ipſius octuplum, nonniſi  $0,96027328$ ; adeoque deest ſaltem  $0,03972672$ , (quod *Complementum* appello) ad 1, Integrum.

Negleſtis igitur Numeratoribus  $2,3,4,5,6,7,8$ , (ut qui Fractionem Truncatam nihilo accuratiorem exhibituri ſint, quam eſt Truncata prima  $\frac{1}{3}$ ;) Numeratori  $9=1$

1. 3 |  $12003416$

8. 24 |  $96027328$

9. 28 |  $08030744$

ipſi  $3,12003416$ , Denominatori primo, ſuperaddat ipſius Octuplum,  $24,96027328$ ;

Nempe  $28,08030744$ . Qui Fractionem, quam *Secundam* appel-

lo, Completam exhibet  $\frac{9}{28,08030744}$ ; Truncatam  $\frac{9}{28}$ ; hoc

eſt  $\frac{1+8}{3+25}$ : Quanvis & juſto majorem eſſe conſtat, (propter Denominatorem truncatum,) & truncatâ primâ  $\frac{1}{3}$  minorem, per

Lemma noſtrum (propter minorem rationem Augmentorum 8 ad 25, quam Terminorum 1 ad 3,) adeoque propiorem vero. Sed

&, per idem Lemma, major eſt eadem  $\frac{9}{28} = \frac{1+8}{3+25}$ , quam  $\frac{8}{25}$ ;

propter majorem rationem Augmentorum 1 ad 3, quam Terminorum 8 ad 25. Cui ſimile in ſequentibus ſæpius animadvertendum occurret.

Pari ratione; post Fractionem truncatam secundam,  $\frac{2}{3}$ ; negligendi sunt Numeratores septem sequentes, 10, 11, 12, 13, 14, 15, 16; qui Numeratori 9, superaddunt, 1, 2, 3, 4, 5, 6, 7. Quum enim requiratur, ut dictum est, plusquam octuplum Mantissæ primæ quo transferatur 1 ad locum Integrorum; Manifestum est, Mantissæ primæ, septuplum (necum quæ hoc minora sunt multipla) una cum Mantissa Fractionis secundæ 0,08030744 (quam Mantissa prima minorem fore constat id neutiquam præstare. Et propterea (cum nihil ex locis partium Decimalium accedat) Augmenta terminorum Fractionis truncatæ  $\frac{2}{3}$ , erunt in ratione 1 ad 3; (quippe quoties 1 additur Numeratori 9, toties additur numerus 3 Denominatori 28:) Quæ quidem Augmentorum ratio 1 ad 3, cum major sit ratione Terminorum 9 ad 28; augebitur hac accessione (non minuetur) Fractio  $\frac{2}{3}$ , quæ tamen ipsa est iusto major.

Sumpto autem Numeratore  $17 = 9 + 8$ ; Denominatori (qui Numeratori 9 respondet) 28,08030744, addendum erit Denominatoris primi octuplum, 24,96027328 (quod *Continuum Incrementum* appello:) quorum Mantissæ simul additæ cum superent 1, Integrum (est utique prior, major quàm posterioris Complementum,) transferetur 1 ad sedem Integrorum. Unde habebitur Fractio, quam

|     |    |  |          |
|-----|----|--|----------|
| 9.  | 28 |  | 08030744 |
| 10. | 31 |  | 20034160 |
| 11. | 34 |  | 32037576 |
| 12. | 37 |  | 44040992 |

&c. &c.

Tertiam voco, Completa,  $\frac{17}{53,04058072}$ ; Truncata vero  $\frac{17}{53}$ ;

hoc est,  $\frac{9+8}{28+25}$ : Quæ tum iusto major est (propter Denominatorem Truncatum,) tum præcedente  $\frac{2}{3}$  minor (per Lemma nostrum,) adeoque vero propior. Sed & per idem Lemma, major quàm  $\frac{2}{3}$ .

Atque hoc eousque repetendum erit (eisdem de causis) quamdiu proxime repertæ Fractionis Mantissa tanta fuerit, ut octuplo Mantissæ primæ addita, transferat 1 ad Integros: Hoc est, quamdiu proveniens Mantissa saltem non minor fuerit istius octupli (seu continui Incrementi) complemento supra dicto 0,03972672.



$$\begin{array}{r|l} 17.53 & 04058072 \\ 8.24 & 96027328 \\ \hline 25.78 & 00085400 \end{array}$$

Adeoque, post Fractionem Tertiam  $\frac{17}{25}$ ; neglectis proxime sequentibus Numeratoribus septem, nempe 18, 19, 20, 21, 22, 23, 24, (tanquam inutilibus, ob causas jam traditas;) sequenti 25 = 17 + 8, Denominatorem ut prius apto (suppetit utique Fractioni tertiae, Mantissa Complemento octupli Primae major.) Eritque Fractio, quam *Quartam* appello,

Completa  $\frac{25}{78,00085400}$ ; Truncata,  $\frac{25}{78} = \frac{17+8}{53+25}$ ; Quæ tum iusto major est (ob Denominatorem truncatum,) tum præcedente  $\frac{17}{25}$  minor, per Lemma nostrum, (propter Augmentorum rationem 8 ad 25, minorem ratione Terminorum 17 ad 53, ut dictum est;) adeoque vero propior. Sed &c, per idem Lemma, major quam  $\frac{17}{25}$ .

Si autem eo devenum fuerit, ut Mantissa Fractionis proximè repertæ minor fuerit dicto complemento octupli primæ; huic Fractionum ordini terminus imponitur. Quippe jam, non Octavo quidem, sed Nono demum loco, transibit ad integros; quo Truncatæ Fractionis Denominator augeatur.

Adeoque, cum Fractionis Quartæ (primi ordinis) Mantissa 0,00085400, minor sit, necessario complemento 0,03972672; adeoque 0,96112728 Mantissæ octupli Fractionis primæ addita non efficiat 1 integrum, ad sedem integrorum transfere-  
rendum: Hæc *Quarta* Fractio, erit *Primi Ordinis Ultima*. Quippe jam, non modo (ut prius) qui sequuntur Numeratores septem (26, 27, 28, 29, 30, 31, 32,) inutiles erunt; sed nec, qui octavo loco sequitur, 33 utilis erit.

$$\begin{array}{r|l} 25.78 & 00085400 \\ 8.24 & 96027328 \\ \hline 33.102 & 96112728 \\ 1.3 & 12002416 \\ \hline 34.106 & 08110144 \end{array}$$

Prodiret utique Fractio completa  $\frac{33}{102,96112728}$ ; Truncata,  $\frac{33}{102} = \frac{11}{34}$ ; hoc est  $\frac{25+8}{78+24}$ . Quæ itaque (cum ratio augmentorum 8 ad 24, hoc est, 1 ad 3, major sit ratione terminorum 25 ad 78,) major est (adeoque à vero remotior) quam præcedens  $\frac{17}{25}$ .

Verum quidem est, in loco sequente, sumpto Numeratore 34, transiturum esse 1 ad numeros integros. Erit utique Fractio

completa  $\frac{34}{106,08116144}$ ; Truncata,  $\frac{34}{106}$

hoc est,  $\frac{25+9}{78+28}$ . Sed hic, cum ratio aug-

mentorum 9 ad 28 ( eadem cum ratione terminorum (Fractio-  
nis secundæ) major sit ratione terminorum (Fractio-  
nis Quartæ) 25 ad 78; major est & Fractio  $\frac{25}{78}$  ( adeoque à vero remotior )  
quam  $\frac{25}{78}$ ; (estque ejusdem plane valoris cum Fractio-  
ne tertia  $\frac{1}{3}$ ) Adeoque proposito non convenit.

Eodem modo ostendetur, neque utilem esse Numeratorem  $42=34+8=25+17$ ; ubi iterum transferetur 1 ad numeros integros. Qui Fractio-  
nem Truncatam exhi-

bet  $\frac{42}{131} = \frac{25+17}{78+53}$ ; majorem ipsâ  $\frac{25}{78}$ ,

propter rationem Augmentorum 17 ad 53 ( eandem cum ratio-  
ne terminorum Fractio-  
nis tertiæ ) majorem ratione Terminorum  
25 ad 78.

Neque ulla prodibit truncata Fractio, post  $\frac{25}{78}$ , usque ad  $\frac{50}{156} =$   
 $\frac{25+25}{78+78}$ ; quæ non sit major ipsâ  $\frac{25}{78}$ ; adeoque à vero remo-  
tior.

Erit autem quæ Numeratori  $50=25+25$  convenit Fractio truncata, non ma-  
jor quidem quam  $\frac{25}{78}$ ; at neque minor; sed  
ejusdem planè valoris, ( propter terminorum Augmenta ipsis terminis proportionalia.) Majorem tamen  
Mantissam habet, nempe 0,00170800: sed non tantam ut Man-  
tissæ primæ octuplo 0,96027328 addita conficiat 1 integrum,  
(utpote hujus complemento 0,03972672 minorem;) adeoque  
nec octavo post hunc loco, transibit 1 ad integros; sed saltem lo-  
co nono, ut post Numeratorem 25.

Post Numeratorem igitur 50, usque ad  $75=50+25=25+25+25$ , nullam prodire Fractio-  
nem truncatam, quæ non sit ma-  
jor quam  $\frac{25}{78}$ , seu  $\frac{1}{3}$ ; eodem modo ostendetur, quo de Nume-  
ratoribus

$$\begin{array}{r} 25, 78 | 00085400 \\ 9, 28 | 78030744 \end{array}$$

$$\begin{array}{r} 34, 106 | 08116144 \\ 8, 24 | 96027328 \end{array}$$

$$42, 131 | 04143472$$

$$\begin{array}{r} 25, 78 | 00085400 \\ 17, 53 | 24058072 \end{array}$$

$$\begin{array}{r} 42, 131 | 04143472 \\ 8, 24 | 96027328 \end{array}$$

$$50, 156 | 00170800$$

$$\begin{array}{r} 25, 78 | 00085400 \\ 25, 78 | 00085400 \\ 50, 156 | 00170800 \end{array}$$

ratoribus qui ipsis 25 & 50 interjecti sunt ostenditur: Et similiter in sequentibus intervallis.

Sumpto autem Numeratore  $75 = 50 + 25 = 25 \times 3$ ; Eadem iterum prodit truncata Fractio; Nempe  $\frac{75}{34} = \frac{25}{78}$ : Sed cum majore adhuc Mantissa (nempe tripla Mantissæ Fractionis quartæ) sed quæ nondum sufficit, (minor utique est complemento necessario c, 03972672.)

Et similiter de Numeratore  $100 = 75 + 25$  dicendum erit. Et sic deinceps, additis continue 25 Numeratori præcedenti; donec tandem suppetit Mantissa, quæ non sit minor necessario complemento c, 03972672; ut nempe octuplo Mantissæ primæ addita compleat 1 integrum. Quippe tum, octavo deinceps loco transibit 1 ex partibus Decimalibus ad sedem integrorum.

Quando autem hoc continget; Divisione inquiritur. Diviso itaque (complemento illo) c, 03972672, per c, 0008400 (mantissam Fractionis quartæ  $\frac{2}{8}$ ) habetur Quotiens 45, 51 —. Fractionis

itaque  $\frac{25}{78,0008400}$  (quam *Primæ Ordinis ultimam* modò diximus, atque *Ordinis Secundi primam* jam constitui-  
mus,) utrique Terminò, addendus

erit sui Multiplus per 46. Nempe, Numeratori 25, numerus 1150; & Dénominat. 78,0008400, numerus 3588,03978400.

Unde habetur Fractio completa,  $\frac{1175}{3666,04013800}$ ; Truncata,  $\frac{1175}{3666}$ ; ejusdem quidem valoris cum  $\frac{2}{8}$ , sed cum majore Mantissa, c, 4013800; quæ cum major sit quam c, 03972672; eadem Mantissæ primæ Octuplo addita conficiet 1 integrum.

Sumpto itaque Numeratore 1183 = 1175 + 8 = 25 + 1158; habebitur ei correspondens Denominator 3691, 00041128. Qui Fractionem (quam *Secundi Ordinis secundam* appello) com-

pletam exhibet  $\frac{1183}{3691,20041128}$ ; Truncatam,  $\frac{1183}{3691}$ ; hoc est,

$\frac{1175}{3666} + \frac{8}{25}$ , seu,  $\frac{25}{78} + \frac{1158}{3613}$ . Quæ tum iusto major est, (pro-

pter Denominatorem truncatum; ) tum minor quàm  $\frac{1175}{3666} = \frac{25}{78}$   
(propter rationem augmentorum 8 ad 25, majorem ratione terminorum 1175 ad 3666, seu 25 ad 78, ut supra dictum est; adeoque & augmentorum 1158 ad 3613, majorem ratione terminorum 25 ad 78: ) major autem quàm  $\frac{1158}{3613}$ .

Atque si huic Fractioni Mantissa tanta suppeteret, ut Mantissæ 0,99955728 addita conficeret i integrum: idem repetendo, à Fractione hac secunda ad tertiam hujus Ordinis procedendum, (additis nempe 1158 Numeratori, & 3612, 99955728 Denominatori; quod voco *Continuum Incrementum Ordinis Secundi* ) quippe tum in Fractione truncata, Augmentorum ratio esset 1158 ad 3613, quæ (ut dictum est) minor est ratione terminorum 1183 ad 3691; adeoque minueretur Fractio. Et sic deinceps quamdiu sufficiens Mantissa superesset.

Quoniam autem Mantissa 0,00041128, minor 0,00041128 est quam ut hoc præstet (esset utique saltem non minor quàm 0,00041272 ) Secunda hæc Fractio est  $\frac{0,99955728}{0,99996856}$  )  
*Secundi Ordinis ultima*, eademque (si inquisitionem ulterius prosequi libeat) erit *Tertii Ordinis prima*, cui *Secunda* aptanda erit eodem modo quo in Secundo Ordine. Et sic deinceps quousque opus erit.

Sed cum Fractio jam ultimò reperta  $\frac{1181}{3691}$ , Denominatorem habeat imperato longè majorem; neque post  $\frac{1}{18}$ , ante hanc, alia vero propior occurrat: Patet ipsam  $\frac{1}{18}$  fractionem imperatam esse; quæ nempe vero *proxime major* sit Denominatorem habens numero 999 non majorem; & quidem in terminis minimis; quippe si in minoribus adhuc terminis, ejusdem valoris Fractio haberi posset, ea prius occurrisset.

Fractionem *proximè minorem datâ*, cujus Denominator numerum datum non excedat; eâdem planè methodo, quâ proximè majorem, inquiri: Nisi quòd jam Numeratorem per Denominatorem dividam; quo habeam, in partibus Decimalibus, Numeratorem Denominatori i convenientem; qui Fractionem, quam *Primam* voco, compleat, expositâ (satis accuratè) æqualem.

Diviso

Diviso itaque Fractionis expositæ  $\frac{2684799}{8376571}$  Numeratore, 2684799, per Denominatorem 8376571; quotientem habeo, in partibus decimalibus, 0,32050931 + satis accuratum; adeoque Fractionem Primam  $\frac{0,32050931}{1}$ ; quam itaque pro exposita substituo, ut ipsi (satis accuratè) æqualem.

| Numerat.     | Denominat. |
|--------------|------------|
| 0   32050931 | 1          |
| 0   96152793 | 3          |
| 1   28203724 | 4          |
| 0   96152793 | 3          |
| 2   24356517 | 7          |
| 0   96152793 | 3          |
| 3   20509310 | 10         |
| 0   96152793 | 3          |
| 4   16662103 | 13         |
| 0   96152793 | 3          |
| 5   12814896 | 16         |
| 0   96152793 | 3          |
| 6   08967689 | 19         |
| 0   96152793 | 3          |
| 7   05120482 | 22         |
| 0   96152793 | 3          |
| 8   01273275 | 25         |

Deinde, (quia Fractionis truncatæ valor non variatur, donec, æqualiter multiplicando utrumque Terminum Completæ, ali- quid ex partibus decimalibus transferatur ad numeros integros;) Dividendo 1, seu 1,00000000, per mantissam 0,32050931; Quotientem habeo 3,12 +. Unde liquet plus quam Triplum requiri, quo transeat 1 ad Integros.

Adeoque (neglectis Denominatoribus 1, 3,) Terminis Fractionis Primæ, addo eorundem respectivè triplum; (quod *Com- num Incrementum* appello;) nempe, Denominatori 1, addo 3, ut habeatur novus Denominator  $4 = 1 + 3$ ; & Numeratori 0,32050931, addo hujus triplum 0,96152793 ut habeatur novus Numerator. Adeoque habetur Fractio *Secunda*, Completa  $\frac{1,28203724}{4}$ ; Truncata  $\frac{1}{4} = \frac{0+1}{1+3}$ : Quæ quidem iusto minor est, (propter truncatum Numeratorem) sed major truncata prima  $\frac{2}{3}$ ; adeoque vero propior: Minor tamen quam  $\frac{1}{3}$ .

Hujusque secundi Numeratoris Mantissa, cum major sit quam 0,03847207 complementum Tripli Mantissæ primæ, seu continui Incrementi; adeoque huic Triplo addita, constituat saltem 1 Integrum: Idem repeto, Nempe, secundo Denominatori 4, addo Triplum primi 3; & secundo Numeratori 1,28203724, addo primi Triplum, 0,96152793; unde prodit Fractio *Tertia*, completa,  $\frac{2,24356517}{7}$ ; Truncata  $\frac{2}{7} = \frac{1+1}{4+3}$ . Quæ iusto quidem minor est, sed major præcedente  $\frac{1}{4}$ ; adeoque vero propior. Sed minor quam  $\frac{1}{3}$ . Cumque

Cumque adhuc suppetit sufficiens Mantissa; idem repeto. Proditq; Fractio *Quarta* completa  $\frac{320509310}{10}$ ; Truncata  $\frac{31}{10}$  iusto quidem minor; sed maior præcedente  $\frac{2}{7}$ .

Idemque sæpius repetendo, obtineo sequentes Fractiones *Quintam*, *Sextam*, *Septimam*, *Octavam*, *Nonam*; Truncatas,  $\frac{7}{10}$ ,  $\frac{13}{10}$ ,  $\frac{16}{10}$ ,  $\frac{19}{10}$ ,  $\frac{22}{10}$ ,  $\frac{25}{10}$ ; continuè iusto propiores.

Quoniam verò, quæ *Nona* superest mantissa c, 01273275, minor est quàm ut triplo Mantissæ primæ addita constituat integer; (utique ipsius complemento c, 03847207 minor:) Fractionem hanc *Nonam* (ob causas superius traditas) concludo *Ordinis Primi ultimam* esse; eandemque *Primam Ordinis secundæ*.

Et quoniam (ut ex supra traditis colligitur) post hanc Ordinis Primi Fractionem ultimam  $\frac{2}{7}$ ; non alia occurrat quæ non sit à vero

remotior, usque ad  $\frac{16}{50} = \frac{8}{25} = \frac{8}{25}$

quæ est ipsi  $\frac{2}{7}$  æqualis; sed cum maiore Mantissa, nempe prioris duplâ: Et sic deinceps, per intervalla, donec tandem provenit Mantissa quæ minor non sit debito complemento c, 03847207: Quæro, quoties repetenda erit illa Mantissa c, 01273275, hujus Ordinis prima, ut complementum illud æquet superetve.

Cumq; Complem. c, 03847207 per mantissam c, 01273275, dividendo, Quotientem habeo 3,02 + video plusquam Triplum requiri.

Ideoq; Fractionis,  $\frac{8,01273275}{25}$ ,

utrique Terminò, sui Triplum addo: habeoq; Fract.  $\frac{32,05093100}{100}$ ,

ipsi quidem æqualem, & cum suffi-

|      |          |      |
|------|----------|------|
| 8    | 01273275 | 25   |
| 8    | 01273275 | 25   |
| 16   | 02546550 | 50   |
| 8    | 01273275 | 25   |
| 24   | 03819825 | 75   |
| etc. |          | etc. |

|    |          |     |
|----|----------|-----|
| 8  | 01273275 | 25  |
| 24 | 03819825 | 75  |
| 32 | 05093100 | 100 |
| 0  | 96152793 | 3   |
| 33 | 01245093 | 103 |
| 24 | 03819825 | 75  |
| 0  | 96152793 | 3   |
| 24 | 99972618 | 78  |
| 8  | 0273275  | 25  |
| 24 | 99972618 | 78  |
| 33 | 01245093 | 103 |
| 24 | 99972618 | 78  |
| 58 | 01218511 | 181 |
| 24 | 99972618 | 78  |
| 83 | 01191129 | 259 |
| 24 | 99972618 | 78  |

|     |          |      |
|-----|----------|------|
| 108 | 01163747 | 317  |
| 24  | 99972618 | 78   |
| 133 | 01130365 | 415  |
| 24  | 99972618 | 78   |
| 158 | 01108983 | 493  |
| 24  | 99972618 | 78   |
| 183 | 01081601 | 571  |
| 24  | 99972618 | 78   |
| 208 | 01054219 | 649  |
| 24  | 99972618 | 78   |
| 233 | 01026837 | 717  |
| 24  | 99972618 | 78   |
| 258 | 00999455 | 805  |
| 24  | 99972618 | 78   |
| 283 | 00972073 | 883  |
| 24  | 99972618 | 78   |
| 308 | 00944691 | 961  |
| 24  | 99972618 | 78   |
| 333 | 00917309 | 1039 |
| 24  | 99972618 | 78   |
| 358 | 00889927 | 1117 |
| 24  | 99972618 | 78   |
| 383 | 00862545 | 1195 |
| 24  | 99972618 | 78   |
| 408 | 00835163 | 1273 |
| 24  | 99972618 | 78   |
| 433 | 00807781 | 1351 |
| 24  | 99972618 | 78   |
| 458 | 00780399 | 1429 |
| 24  | 99972618 | 78   |
| 483 | 00753017 | 1507 |
| 24  | 99972618 | 78   |
| 508 | 00725635 | 1585 |
| 24  | 99972618 | 78   |
| 533 | 00698253 | 1663 |
| 24  | 99972618 | 78   |

ciente Mantissa; quæ nempe Mantissæ primæ primi ordinis Triplo addita, conficiat i integrum.

Sumpto itaque Denominatore  $109 = 100 + 9 = 25 + 78$ ; huic respondet Numerat. 33, 01245893 Adeoque Fractio (quam *Secundi Ordinis secundam* appello) Completa  $33, 01245893$ ; Truncata 103,

$\frac{33}{103}$ ; hoc est  $\frac{32+1}{100+3}$  seu  $\frac{8+25}{25+78}$ .

Quæ tum justo minor est (propter Numeratorem truncatum),

tum major quam  $\frac{32}{100} = \frac{8}{25}$  (propter

rationem Augmentorum i ad 3, maiorem ratione terminorum 32 ad 100, seu 8 ad 25; adeoque & rationem Augmentorum 25 ad 78, maiorem ratione Terminorum 8 ad 25;) adeoque vero propior. Minor autem quam

$\frac{25}{78}$ .

Iterumq; (quia sufficiens Mantissa suppetit;) additis continuè eisdem Augmentis; (seu *Ordinis Secundi continuo Incremento*) nempe Denominatori 78, & Numeratori 24, 99972618; habentur Fractiones huius Ordinis *Tertia, Quarta, Quinta*, &c. & sic deinceps, longa serie, quandiu suppetit sufficiens Mantissa; hoc est, quæ minor non sit quam 0,00027387, complem. Mantissæ 0,99972618 ad i integrum. Quæ quidem Fractiones Truncatæ, continuè propior





|      |          |      |
|------|----------|------|
| 1008 | 00177995 | 3145 |
| 24   | 99972618 | 78   |
| 1033 | 00150613 | 3223 |
| 24   | 99972618 | 78   |
| 1058 | 00123231 | 3301 |
| 24   | 99972618 | 78   |
| 1083 | 00095049 | 3379 |
| 24   | 99972618 | 78   |
| 1108 | 00068407 | 3457 |
| 24   | 99972618 | 78   |
| 1133 | 00041085 | 3535 |
| 24   | 99972618 | 78   |
| 1158 | 00013703 | 3613 |
| 24   | 99972618 | 78   |
| 1182 | 99986321 | 3691 |
| 1158 | 00013703 | 3613 |
| 1182 | 99986321 | 3691 |
| 2341 | 00000024 | 7304 |

sic deinceps quamdiu suppetit Mantissa sufficiens, nempe quæ minor non sit complemento Mantissæ continui Incrementi hujus Ordinis. Ubi autem non suppetit mantissa sufficiens; habetur hujus Ordinis Fractio Ultima, eademque Prima sequentis. Et sic deinceps quousque opus fuerit.

Quoniam autem (in casu præfenti) ubi ad Ordinis secundi Fractionem decimam quartam,  $\frac{1111}{1019}$ , perventum est, habetur Denominator imperato major (ut qui tres locos excedit;) concludo, proximè præcedentem,  $\frac{104}{61}$ , Fractionem expositâ proximè minorem esse, cujus Denominator datum numerum 999 non excedit; & quidem in terminis minimis.

Quodque in hac Fractione expositâ factum est; in alia quavis, mutatis mutandis, peragendum erit.

*Summa Præceptorum huc redit.*

Si quæretur Fractio, Denominatorem habens dato Numero non majorem, quæ sit expositâ Fractione proximè major; Dividatur expositæ Denominator per Numeratorem: Si, quæ proximè minor; Numerator per Denominatorem: Ut habeatur Quotiens, per partes Decimales continuatus, satis accuratus. Qui quidem Quotiens, ubi proximè Major quæritur, sit Denominator, Numeratori respondens; ubi proximè Minor, sit Numerator, respondens Denominatori 1: Fractionem expositæ satis accuratè æqualem, complens. Quam, Fractionem Primam completam, appello: Eandemque, Mantissa Partium Decimalium mulctatam, appello, Fractionem primam Truncatam.

Deinde, per Mantissam hanc Fractionis Primæ, divido 1 integrum: Et per Numerum integrum, Quotiente illius Divisionis accurato proximè minorem (intellige, Quotientem adhærente

rente Fractione, seu Mantissa partium Decimalium, si quam habet, multiplicatum; vel, si nullam habet, multiplicatum integrâ unitate; & sic alibi:) Multiplico Fractionis primæ completæ utrumque Terminum, tum Numeratorem scilicet, tum Denominatorem: Factumque hac multiplicatione numerum respectivè, appello, *Continuum Incrementum* eorundem respectivè Terminorum. Quodque Mantissæ partium decimalium, huic Incremento continuo adherenti, deest ad 1 Integrum, appello Mantissæ continui Incrementi *Complementum*.

Aucto deinde tum Numeratore tum Denominatore Fractionis Primæ, suo respectivè continuo Incremento; habentur Termini *Fractionis Secundæ*: Atque hi similiter iisdem continuis Incrementis aucti, Terminos exhibent Fractionis *Tertiæ*; & sic deinceps; quamdiu suppetit provenienti Fractioni Mantissa quæ minor non sit quam Mantissæ continui Incrementi complementum.

Quamprimum autem Mantissa sic proveniens, sit illo complemento minor; Fractionem cui illa convenit Mantissa minor, constituo, *Primi Ordinis ultimam*; eandemque *Secundi primam*.

Per hujus autem Fractionis (ordinis Secundi primæ) *Mantissam*; divido *Complementum* illud Mantissæ continui Incrementi Ordinis præcedentis; Et, per Numerum integrum accurato Quotiente proxime minorem, multiplico Fractionis primæ hujus Ordinis utrumque Terminum: Factumque respectivè numerum continuo incremento Ordinis proximè præcedentis respectivè auctum, constituo hujus *secundi Ordinis*, respectivè *continuum Incrementum*: Quodque hujus secundi Incrementi continui Mantissæ deest ad 1 Integrum, appello (ut prius) *ejusdem Complementum*.

Fractionis demum, hujus secundi Ordinis, primæ terminis, continuo hujus Ordinis Incremento (respectivo) continuè auctis; habentur, successive, Termini Fractionis, hujus Ordinis, *Secundæ, Tertiæ*, & sic deinceps; quamdiu scilicet suppetit Mantissa sufficiens, (quæ nempe complemento Mantissæ continui Incrementi hujus Ordinis non sit minor :) Ubi autem Mantissa sufficiens primum deficit, est illa *Fraçtio præsentis Ordinis ultima, & prima sequentis*.

Et sic deinceps, quousque opus fuerit. Factis scilicet (ut dictum est) cujusque Ordinis continuis Incrementis, ex Terminorum Fractionis istius Ordinis primæ Multiplis, respectivo Ordinis præcedentis continuo incremento auctis; singulisque ordi-

bus

bus eousque continuatis quamdiu Mantissa suppetit complemento Mantissæ continui incrementi sui non minor: Adeoque, cum minor provenit, minor hæc Mantissa dividens complementum illud, indicat Quotiente suo (adhærentibus partibus si quæ sint, vel si nullæ sint, unitate moltiplicato) Quam multipulum terminorum Fractionis primæ sequentis Ordinis, continuo hujus Incremento audum, futurum sit sequentis Ordinis continuum Incrementum.

Fractiones autem sic inventæ, (quas dico *Primam, Secundam, Tertiam, &c.* Ordinis primi, secundi, tertii, & sic deinceps;) Mantissa partium decimalium moltiplicatæ, (quas itaque *Truncatas* appello;) magis magisque ad justum expositæ Fractionis valorem continue appropinquant: earumque singulæ vel proximè majores vel proximè minores (ut dictum est) omnium non majoribus terminis scriptarum. Ex quibus si seligatur ea quæ Denominatorem habet maximum qui dato numero major non sit: habetur quæsitum.

Quodque de Denominatore datum numerum non excedente, dictum est; de Numeratore datum numerum non excedente non minus verum erit. Totusque processus non minus congruit expositis Fractionibus impropriis (quæ Numeratorem habent Denominatore majorem,) quam propriis Fractionibus. Aut etiam Rationis cujusvis Terminis.

*Exemplum aliud.*

*Fraçtio exposita.*

$$\begin{array}{r} 1152263 \\ 3292181 \end{array} \quad \begin{array}{l} 1152263)3292181(2.857143725+ \\ 3292181)1152263(0.349999894- \end{array}$$

*Pro justo majoribus.*

|                              |                          | Nume-<br>ratores. | Denominatores<br>Truncati. | Mantissa.                 |
|------------------------------|--------------------------|-------------------|----------------------------|---------------------------|
| <i>Communus Incrementum.</i> |                          | 1                 | 2                          | 857143725                 |
| 1.                           | 2,857143725.             | 2                 | 5                          | 714287450                 |
|                              |                          | 3                 | 8                          | 571431175                 |
|                              |                          | 4                 | 11                         | 428574900                 |
|                              |                          | 5                 | 14                         | 285718625                 |
|                              |                          | 6                 | 17                         | 142862350                 |
| 2.                           | 164606.470302,999997350. | 7                 | 20                         | 000006075                 |
|                              |                          |                   |                            | 164611,470323   000003425 |

Quæsitæ Fraçtio proximè major  $\frac{1}{20}$ .

*Pro*

*Pro iusto minoribus.*

*Commune Incrementum.*

0,699999788. 2.  
6,999997880. 20.

*Numeratores. Denominatores.*  
*Truncati. Mantissa.*

|     |           |     |
|-----|-----------|-----|
| 0   | 349999894 | 1   |
| 1   | 049999682 | 3   |
| 8   | 049997562 | 23  |
| 15  | 049995442 | 43  |
| 22  | 049993322 | 63  |
| 29  | 049991202 | 82  |
| 36  | 049989082 | 103 |
| 43  | 049986962 | 123 |
| 50  | 049984842 | 143 |
| 57  | 049982722 | 163 |
| 64  | 049980602 | 183 |
| 71  | 049978482 | 203 |
| 78  | 049976362 | 223 |
| 85  | 049974242 | 243 |
| 92  | 049972122 | 263 |
| 99  | 049970002 | 283 |
| 106 | 049967882 | 303 |
| 113 | 049965762 | 323 |
| 120 | 049963642 | 343 |
| 127 | 049961522 | 363 |
| 134 | 049959402 | 383 |
| 141 | 049957282 | 403 |
| 148 | 049955162 | 423 |
| 155 | 049953042 | 443 |
| 162 | 049950922 | 463 |
| 169 | 049948802 | 483 |
| 176 | 049946682 | 503 |
| 183 | 049944562 | 523 |
| 190 | 049942442 | 543 |
| 197 | 049940322 | 563 |
| 204 | 049938202 | 583 |
| 211 | 049936082 | 603 |
| 218 | 049933962 | 623 |
| 225 | 049931842 | 643 |
| 232 | 049929722 | 663 |
| 239 | 049927602 | 683 |

*Rationum Reductio.*

| <i>Numeratores.</i> | <i>Denom.</i> | <i>Numeratores.</i> | <i>Denom.</i>  |
|---------------------|---------------|---------------------|----------------|
| 246                 | 049925482     | 703                 | 309            |
| 253                 | 049923362     | 723                 | 316            |
| 260                 | 049911242     | 743                 | 323            |
| 267                 | 049919122     | 763                 | 330            |
| 274                 | 049917002     | 783                 | 337            |
| 281                 | 049914882     | 803                 | 344            |
| 288                 | 049912762     | 823                 | 351            |
| 295                 | 049910642     | 843                 | <i>&amp;c.</i> |
| 302                 | 049908522     | 863                 | <i>&amp;c.</i> |

Quæsitæ Fractio, proximè minor  $\frac{1122}{983}$ .

*Exemplum Tertium.*

Exposita Fractio seu Ratio,  $\frac{112216108}{978141672}$ .

Pro proximè majoribus justo.

6

358256108)978141672(2,7302891+.

| <i>Numera.</i>      | <i>Denominator.</i> |                  |
|---------------------|---------------------|------------------|
| <i>Incr. tores.</i> | <i>Incr. Trunc.</i> | <i>Mantissa.</i> |
| 1                   | 1                   | 2                |
|                     | 2                   | 3                |
|                     | 3                   | 4                |
| 4                   | 7                   | 11               |
|                     | 11                  | 19               |
| 26                  | 37                  | 71               |
|                     | 63                  | 172              |
| 89                  | 152                 | 415              |
|                     | 293                 | 1073             |
| 241                 | 634                 | 2731             |
|                     | 875                 | 2389             |
|                     | <i>&amp;c.</i>      | <i>&amp;c.</i>   |

Quæsitæ,  $\frac{112}{415}$ .

*Pro*

*Pro proxime Minoribus iusto.*

978141672)358256108(0,3662610—.

| Numerat. |        | Mantissa. | Incr.      | Deno-<br>minat. | Incr. |
|----------|--------|-----------|------------|-----------------|-------|
| Incr.    | Trunc. |           |            |                 |       |
| 1        | 0      | 3662610   |            |                 |       |
| 3        | 1      | 0987860   | 0,732524   | 2               |       |
| 11       | 4      | 0288320   | 2,930096   | 8               |       |
|          | 15     | 016742    | 10,987860  | 41              | 30    |
| 63       | 26     | 004602    |            | 71              |       |
|          | 89     | 001666    | 62,997064  | 243             | 172   |
| 152      | 241    | 000396    | 151,998730 | 658             | 415   |
| 875      | 1116   | 000314    | 874,999918 | 3047            | 2589  |

&c.

&c.

Quæsitæ,  $\frac{241}{658}$ .

*Exemplum Quartum, & Compendii Specimen*

Verum non semper opus erit, ubi longiores Ordines occurrunt, singulos continue numeros cum Mantissa sua, prosequi. Cum enim non modo Termini Completi, sed & Truncati, quamdiu idem Ordo manet, sint in Progressione Arithmetica; licebit eisdem uti Compendiis, quæ in Progressionibus Arithmetici-  
cis aliis adhibemus; adeoque per saltus, & remotiora inter-  
valla, ab Ordinis cujusque initio, ad ejusdem exitum, vel ad  
ejusdem illum saltem quem quærimus terminum, properare;  
dummodo ipsi transitus ab Ordine ad Ordinem satis accuratè ob-  
serventur, quo commune Incrementum habeatur.

Quin & in hujusmodi Transitu, ubi divisionem Complementi  
Mantissæ continui Incremēti per termini ultimi istius Ordinis,  
seu primi sequentis Ordinis, Mantissam imperamus: Nonnun-  
quam etiam primo intuitu constabit, vel semel tantum, vel bis  
saltem, hanc in illa contineri, cum residuo forsan aliquo (magno  
an parvo, perinde est,) ut divisione instituenda opus non fuerit,  
(quippe illud jam obvium est quod erat Divisione quærendum:)



& quidem, ubi instituenda erit hujusmodi Divisio, non opus erit ut illam per integræ Mantissæ numeros forte satis longos continuemus; sed ad paucos saltem partium decimalium locos, ut saltem quis sit Quotientis numerus integer innotescat; & utrum ille numerus integer Quotientem perfectè absolvat, an aliquod supersit residuum: Quippe hæc tantum illâ divisione quæ-

Sic, verbi gratia, expositâ Fractione  $\frac{121441}{3118761}$ ; Quò Fractionem Truncatam justo majorem consequar; Denominatorem (ciphris aliquam multis per partium decimalium loca continuatum) per Numeratorem divido: Unde Quotientem habeo 1, 334805—; vel (accuratiùs) 1, 3348048886+. Quam quidem divisionem eò saltem usque prosequor, donec de sexto loco partium decimalium securus sum; (aut etiam de septimo vel & octavo:) si Fractioni quærendæ permittendum velim Denominatorem quemlibet numero 999 non majorem. Cum enim Fractionis primæ Mantissa, per numerum plus minus millenarium multiplicanda fuerit; quod in Fractionis primæ loco decimalium sexto à justo aberratum est, idem millecuplatum similiter afficiet locum tertium; fietque forsan non contemnendus error, saltem si non procul à transitu ab Ordine ad Ordinem contingat.

Cumque per hanc Divisionem, obtinuerim Fractionem primam

primi Ordinis,  $\frac{1}{1,334805}$ , vel (accuratiùs)  $\frac{1}{1,3348048886}$ ;

Manifestum est jam primo statim intuitu, Mantissam 0, 3348, (triente unius integri paulo majorem,) bis quidem cum residuo satis amplo, sed non ter contineri in uno integro: Adeoque continuum incrementum Ordinis primi, duplum esse singulorum re-

$$\begin{array}{r} 1, \quad 1 \mid 334805 \\ 2, \quad 2 \mid 669610 \\ \hline 3, \quad 4 \mid 004415 \end{array}$$

$$\begin{array}{r} 1, \quad 1 \mid 3348048886 \\ 2, \quad 2 \mid 6696097772 \\ \hline 3, \quad 4 \mid 0044146658 \end{array}$$

$$3, \quad 4 \mid 0044146658$$

respectivè terminorum primæ Fractionis; nempe, Numeratoris, 2; Denominatoris completi, 1, 669610, seu 2, 6696097772; (Truncati vero 3:) Hujusq; Mantissæ Compl. 0, 330390, seu 0, 3303902228. Fractio itaque primi ordinis secunda, erit completa,  $\frac{3}{4,004415}$ , seu

$$\frac{3}{4,0044146658}; \text{ Truncata } \frac{3}{4}.$$

&c,

&, propter Mantissam 0,004415, minorem complemento 0,330390; erit hæc Primi Ordinis Fractio ultima, & Secundi prima.

Deinde, quoniam non statim obviæ est, primo intuitu, quoties in 0,330390, contineatur 0,004415; illum per hunc dividens Quotientem habeo  $\frac{112122}{4415}$  = 74,9 —. Multiplico igitur Fracti-

onis  $\frac{2}{4,004415}$ , seu  $\frac{3}{4,0044146658}$

utrumque terminum per 74; habeoque 222; & 296,326710, seu 296,3266852692: Quibus addo respectivè commune Incrementum præcedens; 2, & 2,669610,

seu 2,6696097772; habeoque secundi Ordinis commune Incrementum, nempe Numeratoris 224;

& Denominatoris 298,996320, seu, 298,9962950464. Atque hoc

commune Incrementum respectivè additum terminis Fractionis secundi Ordinis primæ, 3, & 4,004415,

seu 4,0044146658; exhibet Fractionis secundæ secundi Ordinis, Numeratorem 227; Denomina-

tem completum 303,000735, seu 303,0007097122; Truncatum 303. Hoc est, Fractionem secundam secundi Ordinis completam,

227, seu  $\frac{227}{303,000735}$ , seu  $\frac{227}{303,0007097122}$

Truncatum  $\frac{227}{303}$ .

Idemque commune Incrementum repetendo, Fractionem habemus secundi Ordinis tertiam, si sufficiens Mantissa suppetaret; (& sic deinceps.) Sed quoniam hoc non fit, ut utique Mantissa 0,000735, seu

3.  
274  
222  
+ 2  
224  
4.004415  
74.  
16.017660  
280.30905  
296.326710  
2.669610  
298.996320  
4.0044146658  
74.  
16.0176586632  
280.309026606  
296.3266852692  
2.6696097772  
298.9962950464  
3. 4 | 004415  
224. 298 | 996320  
227.103 | 000735.  
F 2

$$\begin{array}{r|l} 4 & 0044146658 \\ 198 & 9962950464 \end{array}$$

$$\begin{array}{r} 0,000710, \text{ minor quàm Mantiffa} \\ 0,996320, \text{ seu } 0,996295, \text{ Comple-} \\ \text{mentum } 0,003680 \text{ seu } 0,003705: \end{array}$$

Est ea Fractio, secundi Ordinis ultima, & tertii prima.

Adeoque, quò tertii Ordinis continuum Incrementum habeam per Mantissam  $0,000715$ , divido complementum  $0,003680$ ; vel, accuratius, per Mantissam  $0,000710$ , divido Complementum

$$0,003705 : \text{Habeoque } \frac{0,003680}{0,000735} = \frac{3680}{735} = 5,007-; \text{ seu } \frac{0,003705}{0,000710} = \frac{3705}{710} = 5,2+.$$

(Ubi obiter notandum est, quam necessaria fuerit monitio, de partibus Decimalibus in Fractioe prima saltem ad sextum locum continuandis: Quamquam enim hoc factum fuerit, posito Denominatore primo  $1,334805-$ ; fueritque sexti loci errorculus valde exiguus (cum valor iustus fuerit plusquam  $1,334804886$ ; (prope tamen aberat quin, in Quotientis huius numero integro  $5$ , aberratum fuerit: Cum enim Quotiens accuratus major sit futurus quam  $5,2$ ; minor tamen prodit quam  $5,007$ ; ut ægrè superaverit numerum  $5$  integrum.) Cum itaque Quotientem habeam plusquam  $5$ ; Fractionis proximè inventæ uterque terminus saltem quintuplandus erit; (atq; insuper continuo incremento ordinis præcedentis augendus,) quò habeatur continuum incrementum huius ordinis, terminis Fractionis primæ respective addendum, quò habeantur Fractionis secundæ termini.

Verum Fractionis jam repetæ  $\frac{227}{303}$  Denominator  $303$ , tantus est, ut illius Quintuplus  $1515$  (nedum hic auctus) longè superaturus sit datum numerum  $999$ : Subsisto itaque in Fractioe jam inventa  $\frac{227}{303}$ , ut quæ vero proximè major sit, Denominatorem habens numero dato  $999$  non maiorem.

Pro Fractioe proximè minore, Numeratorem  $1984673$ , per Denominatorem  $5318761$  divido: Unde Quotientem habeo,  $0,749173+$ , vel, accuratius,  $0,749173627-$ : Adeoque Fractionem primam completam  $\frac{0,749173}{1}$ , seu  $\frac{0,749173627}{1}$ ,

Truncatam  $\frac{1}{1}$ .

Cumque

Cumque hujus Mantissa, major sit quam semissis unius integri; adeoque plus quidem quam semel, sed non bis, in 1 Integro contineatur: Erit primi Ordinis commune Incrementum, idemque atque ipsi Fractionis primæ termini, nempe, Numeratoris, 0, 749173; seu 0,7491731627; Denominatoris, 1, Adeoque Fractio secunda, completa,  $\frac{1,498346}{2}$  seu,  $\frac{1,4983463254}{2}$ .

$$\begin{array}{r|rr} 0 & 749173 & 1 \\ 0 & 749173 & 1 \\ \hline 1 & 498346 & 2 \end{array}$$

$$\begin{array}{r|rr} 0 & 7491731627 & 1 \\ 0 & 7491731627 & 1 \\ \hline 1 & 4983463254 & 2 \end{array}$$

Truncata,  $\frac{1}{2}$ . Idemque incrementum iterum additum, exhibet Fractionem tertiam, completam,  $\frac{2,247519}{3}$  seu,  $\frac{2,2475194881}{3}$ .

$$\begin{array}{r|rr} 1 & 498346 & 2 \\ 0 & 749173 & 1 \\ \hline 2 & 247519 & 3 \end{array}$$

Truncatam  $\frac{2}{3}$ . Quæ quidem (propter Mantissam 0,2475, minorem quam Mantissæ communis incrementi complementum, 0,2508,) est primi Ordinis ultima, & secundi prima.

$$\begin{array}{r|rr} 1 & 4983463254 & 2 \\ 0 & 7491731627 & 1 \\ \hline 2 & 2475194881 & 3 \end{array}$$

Cumque Mantissa 0,2475, in complemento 0,2508, semel quidem, sed non bis contineatur; Fractionis hujus Termini semel sumpti, & continuo incremento præcedentis Ordinis respectivè aucti; exhibent secundi Ordinis continuum Incrementum: Numeratoris quidem, 2, 99669, seu 2,9966916508; Denominatoris, 4. Adeoque Fractionem secundi Ordinis secundam, completam,  $\frac{5,244211}{7}$  seu,  $\frac{5,2442112189}{7}$ .

$$\begin{array}{r|rr} 2 & 247519 & 3 \\ 0 & 749173 & 1 \\ \hline 2 & 996692 & 4 \end{array}$$

$$\begin{array}{r|rr} 2 & 2475194881 & 3 \\ 0 & 7491731627 & 1 \\ \hline 2 & 9966926508 & 4 \end{array}$$

Truncatam  $\frac{5}{7}$ .

|   |            |   |
|---|------------|---|
| 2 | 247519     | 3 |
| 2 | 996692     | 4 |
| 5 | 244211     | 7 |
| 2 | 2475194881 | 3 |
| 2 | 9966926508 | 4 |
| 5 | 2442121389 | 7 |

9

Idemque Incrementum continuè addendo, habentur ejusdem Ordinis secundi Fractiones *Tertia, Quarta,*

*Gr.*  $\frac{8,240907}{11,2375974405}$  *Gr.*  $\frac{11,2375974405}{15,2375974405}$  seu  $\frac{8,240907}{11,2375974405}$  *Gr.*  $\frac{11,2375974405}{15,2375974405}$

*Gr.* Truncata vero,  $\frac{11,2375974405}{15,2375974405}$  *Gr.*

Cum hoc jam sæpius iterandum fore, perspicuum sit, (quippe singulis vicibus minuitur Mantissa, numero 0,0033, circiter;) ne longos numeros sæpius continuè addendos habeam, per saltus seu intervalla majora procedo, pro decima quaque Fractione, continui incrementi Decuplum addendo, (possem, pro vigesima quaque vigecupulum posuisse; aut alias pro libitu,) donec Mantissam eo ulque diminutam videam ut non multum superet Mantissæ continui incrementi complementum 0,003308. Hinc ad hujus ordinis Fractionem ultimam, seu sequentis primam, procedo; Puta,  $\frac{224,002775}{299}$ , seu  $\frac{224,002776473}{299}$ , Atque sic deinceps ut opus fuerit.

Nempe Mantissæ continui incrementi complementum 0,003308, divido per Fractionis hujus Ordinis ultimæ, seu sequentis primæ,  $\frac{224,002775}{299}$ ; Mantissam 0,002775; seu potius sine divisione, statim percipio Mantissam hanc in illo complemento, plus quidem quam semel sed non bis contineri: Adeoque Fra-

|     |            |     |
|-----|------------|-----|
| 224 | 002727     | 279 |
| 2   | 996692     | 4   |
| 216 | 999419     | 303 |
| 224 | 0027756473 | 299 |
| 2   | 9966926508 | 4   |
| 226 | 9994682981 | 303 |

Fractionis hujus termini, semel sumpti, autique respectivo præcedentis ordinis continuo incremento, exhibent tertii Ordinis continuum incrementum; Numeratoris quidem 226, 999419, feu 226, 9994682981; Denominatoris, 303. Hujusque continua additione, Fractionis hujus Ordinis reliquas, quousque opus est, reperio, ut ex operatione subiecta patet.

Exposita Fractio,  $\frac{19144211}{5318761}$ .

*Pro proximè majoribus justo.*

3984673) 5318761 (1, 3348048886+.

| Incr. | Numeratores. | Incr. | Denominatores. |                   |
|-------|--------------|-------|----------------|-------------------|
|       | 1            |       | 1              | 334805-           |
| 2     | 3            | 3     | 4              | 004415 2,669610   |
| 224   | 227          | 299   | 303            | 000735 298,996320 |
|       | &c.          |       |                | &c.               |

Vel sic; cum accuratiore Mantissa.

| Incr. | Numeratores. | Incr. | Denominatores. | Incrementum.              |
|-------|--------------|-------|----------------|---------------------------|
|       | 1            |       | 1              | 3348048886+               |
| 2     | 3            | 3     | 4              | 0044146658 2,6696097772   |
| 334   | 227          | 299   | 303            | 0007097122 298,9962950466 |
|       | &c.          |       |                | &c.                       |

Quæsitæ Fractio, proximè major,  $\frac{227}{303}$ .

*Pro*

## Rationum Reductio.

Pro proximè Minoribus.

5318761)3984673 (0.7491731627—

| Increment. | Numerat. | Incr.  | Incr. Denominat. |
|------------|----------|--------|------------------|
|            | 0        | 749173 | 1                |
| 1          | 1        | 498346 | 2                |
|            | 2        | 247519 | 3                |
| 3          | 5        | 244211 | 4                |
|            | 8        | 240903 | 11               |
|            | 11       | 237595 | 15               |
|            | 14       |        | 19               |
|            | 17       |        | 23               |
|            | 20       |        | 27               |
|            | 23       |        | 31               |
|            | 26       |        | 35               |
|            | 29       |        | 39               |
|            | 32       |        | 43               |
|            | 35       |        | 47               |
|            | 38       |        | 51               |
|            | 41       | 204515 | 55               |
|            | 44       |        | 59               |
|            | 47       |        | 63               |
|            | 50       |        | 67               |
|            | 53       |        | 71               |
|            | 56       |        | 75               |
|            | 59       |        | 79               |
|            | 62       |        | 83               |
|            | 65       |        | 87               |
|            | 68       |        | 91               |
|            | 71       | 171435 | 95               |
|            | 74       |        | 99               |
|            | 77       |        | 103              |
|            | 80       |        | 107              |
|            | 83       |        | 111              |
|            | 86       |        | 115              |
|            | 89       |        | 119              |
|            | 92       |        | 123              |
|            | 95       |        | 127              |
|            | 98       |        | 131              |
|            | 101      | 138355 | 135              |
|            | 104      |        | 139              |
|            | 107      |        | 143              |
|            | 110      |        | 147              |



| <i>Increment.</i> | <i>Numerat.</i> | <i>Incr.</i> | <i>Incr. Denom.</i> |
|-------------------|-----------------|--------------|---------------------|
|                   | 113             |              | 151                 |
|                   | 116             |              | 155                 |
|                   | 119             |              | 159                 |
|                   | 122             |              | 163                 |
|                   | 125             |              | 167                 |
|                   | 128             |              | 171                 |
|                   | 131             | 105275       | 175                 |
|                   | 134             |              | 179                 |
|                   | 137             |              | 183                 |
|                   | 140             |              | 187                 |
|                   | 143             |              | 191                 |
|                   | 146             |              | 195                 |
|                   | 149             |              | 199                 |
|                   | 152             |              | 203                 |
|                   | 155             |              | 207                 |
|                   | 158             |              | 211                 |
|                   | 161             | 072195       | 215                 |
|                   | 164             |              | 219                 |
|                   | 167             |              | 223                 |
|                   | 170             |              | 227                 |
|                   | 173             |              | 231                 |
|                   | 176             |              | 235                 |
|                   | 179             |              | 239                 |
|                   | 182             |              | 243                 |
|                   | 185             |              | 247                 |
|                   | 188             |              | 251                 |
|                   | 191             | 039115       | 255                 |
|                   | 194             |              | 259                 |
|                   | 197             |              | 263                 |
|                   | 200             |              | 267                 |
|                   | 203             |              | 271                 |
|                   | 206             |              | 275                 |
|                   | 209             |              | 279                 |
|                   | 212             |              | 283                 |
|                   | 215             |              | 287                 |
|                   | 218             |              | 291                 |
|                   | 221             | 006035       | 295                 |
|                   | 224             | 002727       | 299                 |
| 227               | 451             | 002146       | 602                 |
|                   | 678             | 001565       | 905                 |

226,999419. 303

G

Vel sic, cum accuratiore Mantissa.

| <i>Incr. Numeratores.</i> |     | <i>Increment.</i> | <i>Denom. Incr.</i> |     |
|---------------------------|-----|-------------------|---------------------|-----|
| 1                         | 0   | 7491731627        | 1                   |     |
|                           | 1   | 4983463254        | 2                   | 1   |
|                           | 2   | 2475194881        | 3                   |     |
| 3                         | 5   | 2442121389        | 7                   | 4   |
|                           | 8   | 2409047897        | 11                  |     |
|                           | 11  | 2375974405        | 15                  |     |
| 30                        | 41  | 2045239485        | 55                  | 40  |
|                           | 71  | 1714504565        | 95                  |     |
|                           | 101 | 1383769645        | 135                 |     |
|                           | 131 | 1053034725        | 175                 |     |
|                           | 161 | 0722299805        | 215                 |     |
|                           | 191 | 0391564885        | 255                 |     |
|                           | 221 | 0060829965        | 295                 |     |
| 3                         | 224 | 0027756473        | 299                 | 4   |
| 217                       | 451 | 0022439454        | 602                 | 303 |
|                           | 678 | 0017122435        | 905                 |     |
| <i>&amp;c.</i>            |     |                   | <i>&amp;c.</i>      |     |

Quæsitæ Fractio, proximè minor,  $\frac{611}{905}$ .

Apposui autem, in hoc exemplo, duplicem Calculum: Alterum per Mantissam sex locorum; alterum per accuratiorem Mantissam, locorum decem: Eo fine, ut ostendam quam necessarium sit Mantissam satis longam à principio assumere; ne scilicet exiguus Errorculus ille in ultima figura necessario primitus admitendus, continuo multiplicatus evadat ita notabilis, atque in figuris antecedentes ita se infinuet, ut error inde subrepat non contemnendus.

Cumque Mantissa sex locorum vix aut ne vix sufficiat dum permittitur quæsitæ Fractiōis Denominator ad Millenarium prope accedere; si Major adhuc permittatur, accuratior adhuc Mantissa à principio sumenda erit.

*Exem-*

*Exemplum Quintum.*

Rationem quam habet Circuli Diameter ad Perimetrum, *Clavius*, *Kulenius*, *Snellius*, alique hujusmodi esse, ostendunt.

*Dia.* 1. 00000,00000,00000,00000,00000,00000,00000.

*Peri.* 3. 14159,26535,89793,23846,26433,83279,50288. +

3. 14159,26535,89793,23846,26433,83279,50289 -

*Put.* 3. 14159,26535,89793,23846,26433,83279,50288  $\frac{1}{2}$ .

*Etq;* 1. 00000,00000,00000,00000,00000,00000,00000.

3. 14159,26535,89793,23846,26433,83279,50288  $\frac{1}{2}$ .

0.21830,98861,83790,67153,77675,16745,02872,4.

1.00000,00000,00000,00000,00000,00000,00000,0.

seu = 0,31830,98861,83790,67153,77675,16745,02872,4.

Eadem (proxime) Ratio (eodem modo quo Fractiones,) ad minimos terminos reducta, ita erit.

Ratio Diametri ad Perimetrum Circuli, vero major, sed continuè decrescens: seu, Perimetri ad Diametrum vero minor, sed continuè crescens: donec intra assignatos limites consistat.

|                | <i>Diam.</i> | <i>Perim.</i> |    | <i>Diam.</i> | <i>Perim.</i> |
|----------------|--------------|---------------|----|--------------|---------------|
| I.             | 1            | 3.1415 &c.    | 43 | 135          |               |
| x 7.           | 7            | 22            | 50 | 157          |               |
| <i>Incres.</i> | 7            | 22            | 57 | 179          |               |
|                | 8            | 25            | 64 | 201          |               |
|                | 15           | 47            | 71 | 223          |               |
|                | 22           | 69            | 78 | 245          |               |
|                | 29           | 91            | 85 | 267          |               |
|                | 36           | 113           | 92 | 289          |               |
|                |              |               | 99 | 311          |               |

G 2

II.

|         | Diam. | Perim. | Diam. | Perim. |
|---------|-------|--------|-------|--------|
| II.     | 106   | 333    | 4061  | 12756  |
| x I, +  |       |        | 4174  | 13113  |
| Incram. | 113   | 355    | 4287  | 13468  |
|         | 219   | 688    | 4400  | 13823  |
|         | 332   | 1043   | 4513  | 14178  |
|         | 445   | 1398   | 4626  | 14533  |
|         | 558   | 1753   | 4739  | 14888  |
|         | 671   | 2108   | 4852  | 15243  |
|         | 784   | 2463   | 4965  | 15598  |
|         | 897   | 2818   | 5078  | 15953  |
|         | 1010  | 3173   | 5191  | 16308  |
|         | 1123  | 3528   | 5304  | 16663  |
|         | 1236  | 3883   | 5417  | 17018  |
|         | 1349  | 4238   | 5530  | 17373  |
|         | 1462  | 4593   | 5643  | 17728  |
|         | 1575  | 4948   | 5756  | 18083  |
|         | 1688  | 5303   | 5869  | 18438  |
|         | 1801  | 5658   | 5982  | 18793  |
|         | 1914  | 6013   | 6095  | 19148  |
|         | 2027  | 6368   | 6208  | 19503  |
|         | 2140  | 6723   | 6321  | 19858  |
|         | 2253  | 7078   | 6434  | 20213  |
|         | 2366  | 7433   | 6547  | 20568  |
|         | 2479  | 7788   | 6660  | 20923  |
|         | 2592  | 8143   | 6773  | 21278  |
|         | 2705  | 8498   | 6886  | 21633  |
|         | 2818  | 8853   | 6999  | 21988  |
|         | 2931  | 9208   | 7112  | 22343  |
|         | 3044  | 9563   | 7225  | 22698  |
|         | 3157  | 9918   | 7338  | 23053  |
|         | 3270  | 10273  | 7451  | 23408  |
|         | 3383  | 10628  | 7564  | 23763  |
|         | 3496  | 10983  | 7677  | 24118  |
|         | 3609  | 11338  | 7790  | 24473  |
|         | 3722  | 11693  | 7903  | 24828  |
|         | 3835  | 12048  | 8016  | 25183  |
|         | 3948  | 12403  | 8129  | 25538  |
|         |       |        | 8242  | 25893  |

| <i>Diam.</i> | <i>Perim.</i> |
|--------------|---------------|
| 8355         | 26248         |
| 8468         | 26603         |
| 8581         | 26958         |
| 8694         | 27313         |
| 8807         | 27668         |
| 8920         | 28023         |
| 9033         | 28378         |
| 9146         | 28733         |
| 9259         | 29088         |
| 9372         | 29443         |
| 9485         | 29798         |
| 9598         | 30153         |
| 9711         | 30508         |
| 9824         | 30863         |
| 9937         | 31218         |
| 10050        | 31573         |
| 10163        | 31928         |
| 10276        | 32283         |
| 10389        | 32638         |
| 10502        | 32993         |
| 10615        | 33348         |
| 10728        | 33703         |
| 10841        | 34058         |
| 10954        | 34413         |
| 11067        | 34768         |
| 11180        | 35123         |
| 11293        | 35478         |
| 11406        | 35833         |
| 11519        | 36188         |
| 11632        | 36543         |
| 11745        | 36898         |
| 11858        | 37253         |
| 11971        | 37608         |
| 12084        | 37963         |
| 12197        | 38318         |
| 12310        | 38673         |
| 12423        | 39028         |
| 12536        | 39383         |

| <i>Diam.</i> | <i>Perim.</i> |
|--------------|---------------|
| 12649        | 39738         |
| 12762        | 40093         |
| 12875        | 40448         |
| 12988        | 40803         |
| 13101        | 41158         |
| 13214        | 41513         |
| 13327        | 41868         |
| 13440        | 42223         |
| 13553        | 42578         |
| 13666        | 42933         |
| 13779        | 43288         |
| 13892        | 43643         |
| 14005        | 43998         |
| 14118        | 44353         |
| 14231        | 44708         |
| 14344        | 45063         |
| 14457        | 45418         |
| 14570        | 45773         |
| 14683        | 46128         |
| 14796        | 46483         |
| 14909        | 46838         |
| 15022        | 47193         |
| 15135        | 47548         |
| 15248        | 47903         |
| 15361        | 48258         |
| 15474        | 48613         |
| 15587        | 48968         |
| 15700        | 49323         |
| 15813        | 49678         |
| 15926        | 50033         |
| 16039        | 50388         |
| 16152        | 50743         |
| 16265        | 51098         |
| 16378        | 51453         |
| 16491        | 51808         |
| 16604        | 52163         |
| 16717        | 52518         |
| 16830        | 52873         |

| <i>Diam.</i> | <i>Perim.</i> | <i>Diam.</i> | <i>Perim.</i> |
|--------------|---------------|--------------|---------------|
| 16943        | 53218         | 21237        | 66718         |
| 17056        | 53583         | 21350        | 67073         |
| 17169        | 53938         | 21463        | 67428         |
| 17282        | 54293         | 21576        | 67783         |
| 17395        | 54648         | 21689        | 68138         |
| 17508        | 55003         | 21802        | 68493         |
| 17621        | 55358         | 21915        | 68848         |
| 17734        | 55713         | 22028        | 69203         |
| 17847        | 56068         | 22141        | 69558         |
| 17960        | 56423         | 22254        | 69913         |
| 18073        | 56778         | 22367        | 70268         |
| 18186        | 57133         | 22480        | 70623         |
| 18299        | 57488         | 22593        | 70978         |
| 18412        | 57843         | 22706        | 71333         |
| 18525        | 58198         | 22819        | 71688         |
| 18638        | 58553         | 22932        | 72043         |
| 18751        | 58908         | 23045        | 72398         |
| 18864        | 59263         | 23158        | 72753         |
| 18977        | 59618         | 23271        | 73108         |
| 19090        | 59973         | 23384        | 73463         |
| 19203        | 60328         | 23497        | 73818         |
| 19316        | 60683         | 23610        | 74173         |
| 19429        | 61038         | 23723        | 74528         |
| 19542        | 61393         | 23836        | 74883         |
| 19655        | 61748         | 23949        | 75238         |
| 19768        | 62103         | 24062        | 75593         |
| 19881        | 62458         | 24175        | 75948         |
| 19994        | 62813         | 24288        | 76303         |
| 20107        | 63168         | 24401        | 76658         |
| 20220        | 63523         | 24514        | 77013         |
| 20333        | 63878         | 24627        | 77368         |
| 20446        | 64233         | 24740        | 77723         |
| 20559        | 64588         | 24853        | 78078         |
| 20672        | 64943         | 24966        | 78433         |
| 20785        | 65298         | 25079        | 78788         |
| 20898        | 65653         | 25192        | 79143         |
| 21011        | 66008         | 25305        | 79498         |
| 21124        | 66363         | 25418        | 79853         |

| <i>Diam.</i> | <i>Perim.</i> |                | <i>Diam.</i> | <i>Perim.</i> |
|--------------|---------------|----------------|--------------|---------------|
| 25531        | 80208         |                | 29825        | 93698         |
| 25644        | 80563         |                | 29938        | 94053         |
| 25757        | 80918         |                | 30051        | 94408         |
| 25870        | 81273         |                | 30164        | 94763         |
| 25983        | 81628         |                | 30277        | 95118         |
| 26096        | 81983         |                | 30390        | 95473         |
| 26209        | 82338         |                | 30503        | 95828         |
| 26322        | 82693         |                | 30616        | 96183         |
| 26435        | 83048         |                | 30729        | 96538         |
| 26548        | 83403         |                | 30842        | 96893         |
| 26661        | 83758         |                | 30955        | 97248         |
| 26774        | 84113         |                | 31068        | 97603         |
| 26887        | 84468         |                | 31181        | 97958         |
| 27000        | 84823         |                | 31294        | 98313         |
| 27113        | 85178         |                | 31407        | 98668         |
| 27226        | 85533         |                | 31520        | 99023         |
| 27339        | 85888         |                | 31633        | 99378         |
| 27452        | 86243         |                | 31746        | 99733         |
| 27565        | 86598         |                | 31859        | 100088        |
| 27678        | 86953         |                | 31972        | 100443        |
| 27791        | 87308         |                | 32085        | 100798        |
| 27904        | 87663         |                | 32198        | 101153        |
| 28017        | 88018         |                | 32311        | 101508        |
| 28130        | 88373         |                | 32424        | 101863        |
| 28243        | 88728         |                | 32537        | 102218        |
| 28356        | 89083         |                | 32650        | 102573        |
| 28469        | 89438         |                | 32763        | 102928        |
| 28582        | 89793         |                | 32876        | 103283        |
| 28695        | 90148         |                | 32989        | 103638        |
| 28808        | 90503         | III.           | 33102        | 103993        |
| 28921        | 90858         | x 1, +         |              |               |
| 29034        | 91213         | <i>Increm.</i> | 33215        | 104348        |
| 29147        | 91568         |                |              |               |
| 29260        | 91923         | IV.            | 66317        | 208341        |
| 29373        | 92278         | x 1, +         |              |               |
| 29486        | 92633         | <i>Increm.</i> | 99532        | 312689        |
| 29599        | 92988         |                |              |               |
| 29712        | 93343         |                | 165849       | 521030        |



V  
 $\times 1, +$   
*Increment.*

VI.  
 $\times 1, +$   
*Increment.*

VII.  
 $\times 2, +$   
*Increment.*

VIII.  
 $\times 1, +$   
*Increment.*

IX.  
 $\times 2, +$   
*Increment.*

| <i>Diam.</i> | <i>Perim.</i> |
|--------------|---------------|
| 265381       | 833719        |
| 364913       | 1146408       |
| 630294       | 1980127       |
| 995207       | 3126535       |
| 1360120      | 4272943       |
| 1725033      | 5419351       |
| 3085153      | 9692294       |
| 4810186      | 15111645      |
| 6535219      | 20530996      |
| 8260252      | 25950347      |
| 9985285      | 31369698      |
| 11710318     | 36789049      |
| 13435351     | 42208400      |
| 15160384     | 47627751      |
| 16885417     | 53047102      |
| 18610450     | 58466453      |
| 20335483     | 63885804      |
| 22060516     | 69305155      |
| 23785549     | 74724506      |
| 25510582     | 80143857      |
| 52746197     | 165707065     |
| 78256779     | 245850922     |
| 131002976    | 411557987     |
| 209159557    | 657408909     |
| 340262731    | 1068966896    |
| 811528438    | 2549491779    |
| 1151791169   | 3618458675    |

X.

|                           | <i>Diam.</i>       | <i>Perim.</i>       |
|---------------------------|--------------------|---------------------|
| X.                        | 1963319607         | 6167950454          |
| × 2, +<br><i>Incram.</i>  | 4738167652         | 14885392687         |
| XI                        | 67014872259        | 21053343141         |
| × 8, +<br><i>Incram.</i>  | 567663097408       | 1783366216531       |
| XII.                      | 574364584667       | 1804419559672       |
| × 1, -<br><i>Incram.</i>  | 1142027682075      | 3587785776203       |
| XIII.                     | 1709690779483      | 5371151992734       |
| × 15, +<br><i>Incram.</i> | 2851718461558      | 8958937768937       |
| XIV.                      | 44485467702853     | 139755218526789     |
| × 13, +<br><i>Incram.</i> | 47337186164411     | 148714156295726     |
| XV                        | 91822653867264     | 288469374822515     |
| × 4, +<br><i>Incram.</i>  | 136308121570117    | 428224593349304     |
| XVI.                      | 1816491048114374   | 5706674932067741    |
| × 2, +<br><i>Incram.</i>  | 1952799169684491   | 6134899525417045    |
| XVII.                     | 9627687726852338   | 30246273033735921   |
| × 6, +<br><i>Incram.</i>  | 11580486896536829  | 36381172559152966   |
| XVIII.                    | 21208174623389167  | 66627445592888887   |
| XIX.                      | 136876735467187340 | 430010946591069243  |
| XX.                       | 158084910090576507 | 496638392183958130  |
| XXI.                      | 294961645557763847 | 926649338775027373  |
| XXII.                     | 431838381024951187 | 1356660285366096616 |
| XXIII.                    | 568715116492138527 | 1785671231957165859 |
| XXIV.                     | 705591851959325867 | 2216682178548235102 |

|         | Diam.               | Perim.              |
|---------|---------------------|---------------------|
| XVII.   | 842468587426513207  | 2646693125139304345 |
| x 1, +  |                     |                     |
| Incram. | 979345322893700547  | 3076704071730373588 |
| "       | 1821813910320213754 | 5723397196869677933 |

*Ratio Diametri ad Perimetrum Circuli vero Minor, sed continuè crescens; seu Perimetri ad Diametrum vero Major, sed continuè decrescens; donec intra assignatos terminos consistat.*

|          | Perim. | Diam.          |
|----------|--------|----------------|
| I.       | I      | 0. 318309, &c. |
| x 3, +   |        |                |
| Incram.  | 3      | I              |
|          | 4      | I              |
|          | 7      | 2              |
|          | 10     | 3              |
|          | 13     | 4              |
|          | 16     | 5              |
|          | 19     | 6              |
| II.      | 22     | 7              |
| x 15, +  |        |                |
| Incram.  | 353    | 106            |
| III.     | 355    | 113            |
| x 292, + |        |                |
| Incram.  | 103993 | 33102          |
| IV.      | 104348 | 33215          |
| x 1, +   |        |                |
| Incram.  | 208341 | 66317          |
| V.       | 312689 | 99532          |
| x 2, +   |        |                |
| Incram.  | 833719 | 265381         |

|                                    | <i>Perim.</i> | <i>Diam.</i> |
|------------------------------------|---------------|--------------|
| VI.                                | 1146408       | 364913       |
| $\times 3, +$<br><i>Increment.</i> | 4172943       | 1360120      |
| VII.                               | 5419351       | 1725033      |
| $\times 14, +$<br><i>Incr.</i>     | 80143857      | 25510582     |
| VIII.                              | 85563208      | 27235615     |
| $\times 1.$<br><i>Increment.</i>   | 165707065     | 52746197     |
| IX.                                | 245850922     | 78256779     |
| $\times 2, +$<br><i>Increment.</i> | 411557987     | 131002976    |
| X.                                 | 1068966896    | 340262731    |
| $\times 2, +$<br><i>Increment.</i> | 1480524883    | 471265707    |
| XI.                                | 2549491779    | 811528438    |
| $\times 1, +$<br><i>Increment.</i> | 6167950454    | 1963319607   |
|                                    | 8717442233    | 2774848045   |
| $\times 1, +$<br><i>Increment.</i> | 14885392687   | 4738167652   |
|                                    | 21053343141   | 6701487259   |
|                                    | 35938735828   | 11439654911  |
|                                    | 56992078969   | 18141142170  |
|                                    | 78045422110   | 24842629429  |
|                                    | 99098765251   | 31544116688  |
|                                    | 120152108392  | 38245603947  |
|                                    | 141205451533  | 44947091206  |
|                                    | 162258794674  | 51648578465  |
|                                    | 183312137815  | 58350065724  |
|                                    | 204365480956  | 65051552983  |
|                                    | 225418824097  | 71753040242  |
|                                    | 246472167238  | 78454527501  |
|                                    | 267525510379  | 85156014760  |
|                                    | 288578853520  | 91857502019  |

Increment.

Perim.

Diam.

|               |              |
|---------------|--------------|
| 309632196651  | 98558989278  |
| 330685539802  | 105260476537 |
| 351738882943  | 111961963796 |
| 372792216084  | 118663451055 |
| 393845569225  | 125364938314 |
| 414898912366  | 132066425573 |
| 435952255507  | 138767912832 |
| 457005598648  | 145469400091 |
| 478058241789  | 152170887350 |
| 499112284930  | 158872374609 |
| 520165628071  | 165573861868 |
| 541218971212  | 172275349127 |
| 562272314353  | 178976836386 |
| 583325657494  | 185678323645 |
| 604379000635  | 192379810904 |
| 625432343776  | 199081298163 |
| 646485686917  | 205782785422 |
| 667539030058  | 212484272681 |
| 688592373199  | 219185759940 |
| 709645716340  | 225887247199 |
| 730699059481  | 232588734458 |
| 751752402622  | 239290212177 |
| 772805745763  | 245991708976 |
| 793859088904  | 252693196235 |
| 814912432045  | 259394683494 |
| 835965775186  | 266096170753 |
| 857019118327  | 272797658012 |
| 878072461468  | 279499145271 |
| 899125804609  | 286200632530 |
| 920179147750  | 292902119789 |
| 941232490891  | 299603607048 |
| 962285834032  | 306305094307 |
| 983339177173  | 313006581566 |
| 1004392520314 | 319708068825 |
| 1025445863455 | 326409556084 |
| 1046499206596 | 333111043343 |
| 1067552549737 | 339812530602 |
| 1088605892878 | 346514017861 |

*Perim.*

*Diam.*

|               |              |
|---------------|--------------|
| 1109659236019 | 353215505120 |
| 1130712579160 | 359916992379 |
| 1151765922301 | 366618479638 |
| 1172819265442 | 373319966897 |
| 1193872608583 | 38021454156  |
| 1214925951724 | 386722941415 |
| 1235979294865 | 393424428674 |
| 1257032638006 | 400125915933 |
| 1278085981147 | 406827403192 |
| 1299139324288 | 413528890451 |
| 1320192667429 | 420230177710 |
| 1341246010570 | 426931864969 |
| 1362299353711 | 433633352228 |
| 1383352696852 | 440334839487 |
| 1404406039993 | 447036326746 |
| 1425459383134 | 453737814005 |
| 1446512726275 | 460439301264 |
| 1467566069416 | 467140788523 |
| 1488619412557 | 473842275782 |
| 1509672755698 | 480543763041 |
| 1530726098839 | 487245250300 |
| 1551779441980 | 493946737559 |
| 1572832785121 | 500648224818 |
| 1593886128262 | 507349712077 |
| 1614939471403 | 514051199336 |
| 1635992814544 | 520752686595 |
| 1657046157685 | 527454173854 |
| 1678099500826 | 534155661113 |
| 1699152843967 | 540857148372 |
| 1720206187108 | 547558635631 |
| 1741259530249 | 554260122890 |
| 1762312873390 | 560961610149 |
| 1783366216531 | 567663097408 |

XII.

x2, +

*Incr.*

3587785776203

1142027682075

|                | <i>Perim.</i>    | <i>Diam.</i>     |
|----------------|------------------|------------------|
| XIII.          | 5371151992734    | 1709690779483    |
| * 1, +         |                  |                  |
| <i>Increm.</i> | 8958937768937    | 2851718461558    |
|                | 14330089761671   | 4561409241041    |
|                | 23289027530608   | 7413127702599    |
|                | 31247965299545   | 10264846164157   |
|                | 41206903068482   | 13116564625715   |
|                | 50165840837419   | 15968283087273   |
|                | 59124778606356   | 18820001548831   |
|                | 68083716375293   | 21671720010389   |
|                | 77042654144230   | 24523438471947   |
|                | 86001591913167   | 27375156933505   |
|                | 94960529682104   | 30226875395063   |
|                | 103919467451041  | 33078593856621   |
|                | 112878405219978  | 35930312318179   |
|                | 121837342988915  | 38782030779737   |
|                | 130796280757852  | 41633749241295   |
|                | 139755218526789  | 44485467702853   |
| XIV.           |                  |                  |
| * 3, +         |                  |                  |
| <i>Increm.</i> | 428224593349304  | 136308121570117  |
|                | 567979811876093  | 180793589172970  |
|                | 996204405225397  | 317101710843087  |
|                | 1424428998574701 | 453409832413204  |
|                | 1852653591924005 | 589717953983321  |
|                | 2280878185273309 | 726026075553438  |
|                | 2709102778622613 | 862334197123555  |
|                | 3137327371971917 | 998642318693672  |
|                | 3565551965321221 | 1134950440163789 |
|                | 3993776558670525 | 1271258561833906 |
|                | 4422001152019829 | 1407566683404023 |
|                | 4850225745369133 | 1543874804974140 |
|                | 5278450338718437 | 1680182926544257 |
|                | 5706674932067741 | 1816491048114374 |
| XV.            |                  |                  |
| * 1, +         |                  |                  |
| <i>Increm.</i> | 6134899525417045 | 1952799169684491 |



|                | <i>Perim.</i>       | <i>Diam.</i>       |
|----------------|---------------------|--------------------|
|                | 11841574457484786   | 3769190217798865   |
|                | 17976473982901831   | 5722089387483356   |
|                | 24111373508318876   | 7674888557167847   |
|                | 30246273033735921   | 9627687726852338   |
| XVI.           |                     |                    |
| * 2, +         |                     |                    |
| <i>Incres.</i> | 66527445592888887   | 21208174623389167  |
|                | 96873718526624808   | 30835862350241505  |
|                | 163501164219513695  | 52044036973630672  |
|                | 230128609812402582  | 73252211597019839  |
|                | 296756055405291469  | 94460385220409006  |
|                | 363383500998180356  | 115668560843798173 |
|                | 430010946591069243  | 136876735467187340 |
| XVII.          |                     |                    |
| * 6, +         |                     |                    |
| <i>Incres.</i> | 2646693125139304345 | 842468587426513207 |
|                | 3076704071730373588 | 979345322893700547 |

Exhibetur autem, utrobique (in utraque scil. inquisitione,) Termini posteriores (nempe qui *Perimetro* sunt analogi in inquisitione priore, quique *Diametro* sunt analogi in posteriore) Mantissæ partium Decimalium truncati: (quippe tædium esset tam vastos numeros toties apponere.) Quam tamen facile est ubivis restituere; sicubi id quispiam desideret. Nempe, in inquisitione priore, si terminus quilibet *Diametro* analogus multiplicetur in primum terminorum *Perimetro* analogorum, puta in 3.14159, 26530, 89793, 23846, 26433, 83279, 50288, 5; (intellige, numerum 3, cum Mantissæ partium decimalium:) habebitur respectivus terminus completus, *Perimetro* analogus; Et similiter, in inquisitione posteriore; si terminus quilibet *Perimetro* analogus multiplicetur in primum terminorum *Diametro* analogorum, hoc est, in 0.31830, 98861, 83790, 67153, 77675, 26745, 02872, 4: habetur respectivus terminus completus, *Diametro* analogus. (Unde etiam apparebit quàm prope quælibet rationum minoribus terminis hic exhibitæ ad rationem propositam appropinquat) Quæ quidem Mantissæ sic inventa, eousque tantum pro accuratâ habenda erit, quatenus errorculus postremæ figuræ, Mantissæ primæ sic multiplicatæ; hac Multiplicatione;

catione auctus) se non insinuat in facti figuras aliquot posteriores.

Reperientur autem utrobique, postremorum terminorum aliquot, intra assignatos limites subsistere: hoc est, qui posita *Diametro* 1; exhibebunt *Perimetrum* majorem quidem quam 3, 14159, 26535, 89793, 23846, 6433, 83279, 50288: minorem verò quam 3, 14159, 26535, 89793, 23846, 6433, 83279, 50289: Utrobique scilicet proximè acceditur ad 3, 14159, 26535, 89793, 23846, 6433, 83279, 50288, 5; qui limitibus est intermedius. (Sed non propterea reputandi erunt rationem *Perimetri* ad *Diametrum* accuratius exhibere quàm utervis limitum: nisi constaret (quod non constat) veritatem ipsam præcisè mediam esse intra positos limites; quippe eò collimat nostra inquisitio, ut eam rationem exhibeamus quamproximè quæ est præcisè media; hoc est, quæ pro figura terminali 8, vel 9, habeat  $8\frac{1}{2}$ , seu 8,5.)

Quæ autem intra assignatos limites consistunt; sunt in inquisitione priore, Rationes posteriores quatuor: in inquisitione posteriore, sola ultima. Si enim in singulis his instituat analogia; ut terminus *Diametro* analogus, ad terminum *Perimetro* analogum; sic 1, ad Quartum: reperietur quartus ille terminus, 3, 14159, &c. sed, in quo, pro figura terminali 8, vel 9, prodibit; illic quidem, plus quam 8, sed minus quàm  $8\frac{1}{2}$ ; hic verò, plusquam  $8\frac{1}{2}$ , sed minus quàm 9. Præcedentes autem in utraque inquisitione omnes, sunt extra limites. Siquis autem de his dubitet, poterit ille sibi fidem facere; modò velit quemlibet terminorum *Perimetro* analogorum (Ciphris quot opus erit in locis decimalibus continuatorum) per respectivum terminum *Diametro* analogum dividere, operationem eousque continuando donec Quotientem satis longum nactus fuerit.

Continua Incrementa (in utraque inquisitione) regulis inclusimus, quo a Rationum terminis distinguantur.

Notandum autem erit, quæ sunt in inquisitione priore continua Incrementa, sunt in altera Terminis Rationum; & vice versa, quæ sunt in posteriore continua Incrementa; eadem, in priore, inter Rationum quæsitarum Terminos habentur.

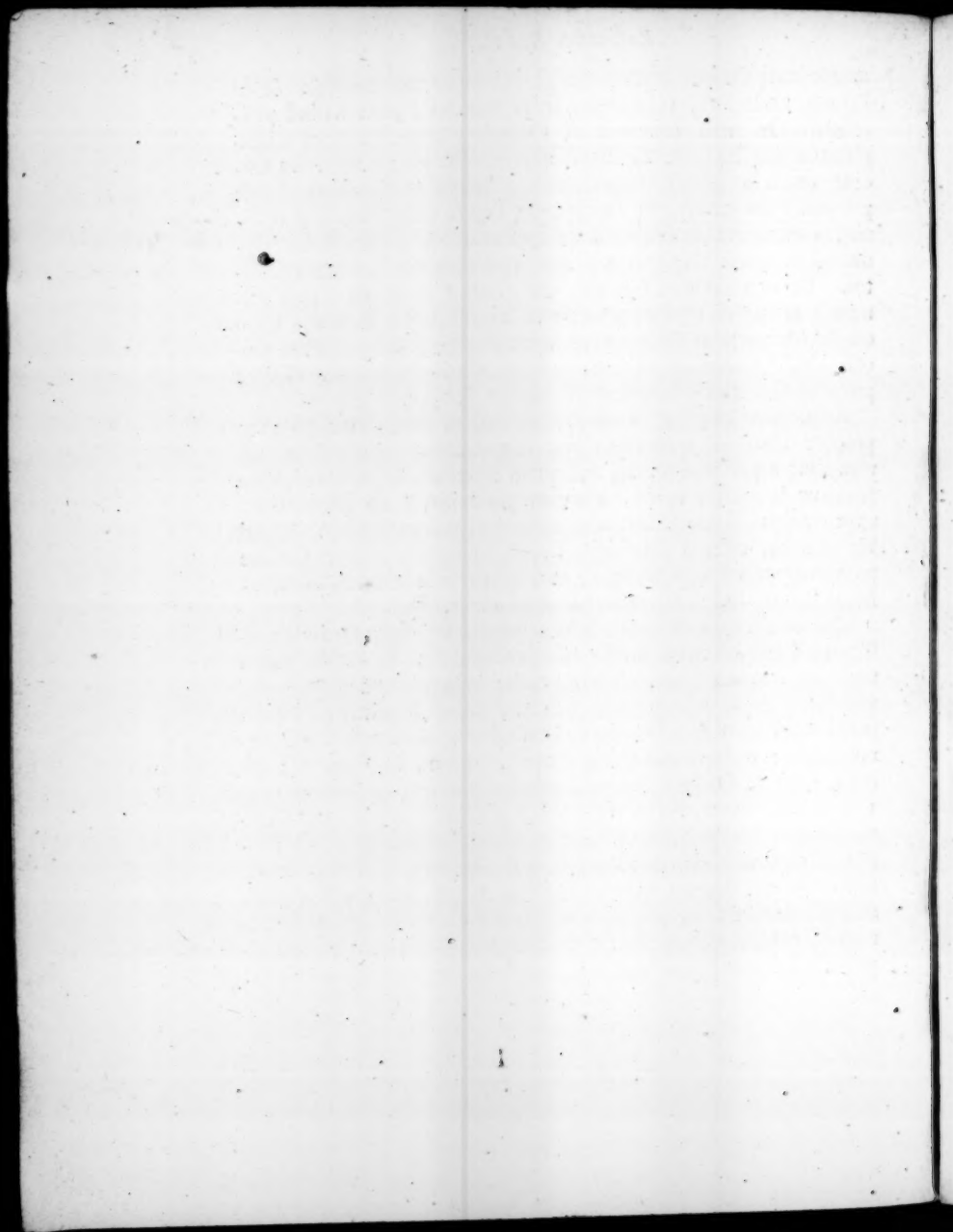
Sed & observare licet, vicissim sibi respondere, Numerum Rationum in quolibet Ordine prioris Inquisitionis; & Multiplicatorem (pro continuo Incremento constituendo) in Ordine correspondente posterioris Inquisitionis: & vice versa, Putà in Ordine undecimo posterioris Inquisitionis, Rationes (post primam, antece-

antecedenti Ordini communem,) habentur numero 84: & in Prioris undecimo, continuum Incrementum (præter illud præcedentis Ordinis) continet sui Ordinis primos terminos multiplicatos per 84: & similiter ubique. Et vice versâ, in Ordine secundo prioris Inquisitionis, habentur Rationes (post primam) numero 292; Ordineque Tertio posterioris Inquisitionis, continuum Incrementum (præter illud præcedentis Ordinis) continet sui Ordinis primos terminos per 292 multiplicatos. Et sic ubique. Nempe; qui *numerat* Rationes (post primam) in quovis Ordine posterioris Inquisitionis; numerus idem est *Multiplicator* in Ordine cognomine Prioris; quique in hoc prioris Ordine *numerat* Rationes, idem est in Posterioris Ordine proximè sequente *Multiplicator*. Et sic semper.

Atque hæc continuè eousque contingere reperiuntur; donec propter Rationes quæ ab origine supponuntur æquales, nec tamen sunt æquales accuratè, sed quàm proximè, (quarum alteram sequitur Inquisitio prior, alteram posterior,) eò perveniatur ut proveniens Ratio sit earum altera major, altera minor: (Quod hic futurum esset si præsentis Inquisitiones uno adhuc Ordine promoveremus.) Quippe tum, non tanquam eadem *utraq;* utrobique Ratio, sed pro diversis habendæ erunt.

Quò autem quæ sit continui Incremēti, cujusque Ordinis constitutio (primo statim conspectu) percipiatur; id ad Marginem breviter insinuaturn est. Verbi gratia, in Inquisitione posteriore, ubi habetur ad Marginem, I. \* 3. *Increm.* innuitur, Ordinis *primi* continuum *Incrementum* factum esse ex primis respectivè terminis istius ordinis *multiplicatis* per 3. Item, II. \* 15, +. *Increm.* innuit, Ordinis *Secundi*, primos terminos *multiplicatos* per 15; atque insuper, respectivè *Addito* continuo Incremento Ordinis proximè præcedentis, exhibere (istius secundi Ordinis) continuum *Incrementum*, quod regulis inclusum ibi inscribitur. Item, III. \* 292 +. *Increm.* innuit, Ordinis *Tertii*, primos Terminos *multiplicatos* per 292, *Addito* præcedentis Ordinis continuo Incremento, exhibere istius Tertii Ordinis *Incrementum* ibidem adscriptum. Et in reliquis similiter.

*Continuum*



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DE  
PERIODO JULIANA  
TRACTATUS.

Sub Anno Æræ Christianæ 1652, (vel non ita multò post,) De Anni numero in *Periodo Juliana* inveni-  
endo, ex datis Numeris in *Cyclis Lunari, Solari*,  
atque *Indictionum*, interrogatus: rem ab origine re-  
petens; atque Exemplis, quàm Præceptis, metho-  
dum inquirendi, ejusque compendia pro re nata,  
feliciter explicatum iri ratus; hanc quæ sequitur  
conscripsi Methodum.

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THE ANNALS OF THE  
FRENCH REVOLUTION  
FROM 1789 TO 1799  
BY  
J. H. M. J. VAN DER  
KAMPE  
LONDON  
1892



## D E

## JULIANA PERIODO.

**E**xposito Anno, qui sit ( verbi gratia ) in Cyclo Solari, Annus, 22; Lunari, 14; Indictionum, 7: quæritur, quotus sit ille Annus Periodi Julianæ?

Cyclus Solaris, est Periodus annorum 28; quo tempore omnes Literarum Dominicalium varietates absolvuntur. Unoque absoluto, alter continuo incipit.

Cyclus Lunaris, est Periodus Annorum 19; quo tempore Lunationum varietates intelliguntur absolvi. Unoque absoluto, alter continuo incipit.

Cyclus Indictionum, est Periodus Annorum 15; ab Indictionibus Romanis ortum habens. Unoque absoluto, alter incipit.

Intelliguntur autem omnes hi Cycli eodem aliquando anno incepisse. Qui annus, cum in singulis illis Cyclis sit primus; primus item habetur Periodi Julianæ ex tribus illis compositæ: quæ tot annorum esse intelligitur, ut, illâ absolutâ, tres illi Cycli simul iterum eodem olim anno sint incepturi.

Quum autem 28, 19, 15, numeri sint inter se primi; fieri non potest, ut, unâ aliquando incipientes, simul terminentur, nisi absolutâ annorum Periodo ab his omnibus composita; nempe annorum  $7980 = 28 \times 19 \times 15$ ; ut qui minimus sit Numerus qui per 28, 19, & 15, dividi possit, per *Prop. 38. El. 7. Euclidis.*

Exposito jam Anno quolibet Periodi Julianæ; puta 6321: Quotus sit ille Annus in singulis illis Cyclis, investigare, facile est. Nam,

Diviso 6321 per 28; Quotiens 225, ostendit Cyclos Solares integros 225 jam transiisse: Et Residuus post divisionem Numerum 21, ostendit annum 21, Cycli jam currentis 226.

x 2

76(1

6321

225

21

Item



Item diviso 6321 per 19; Quotiens 332, ostendit Cyclos Lunares integros 332 jam transiisse; & Residuus 13, indicat sequentis Cycli annum 13.

$$\begin{array}{r} (1 \\ 6321 \\ 6321 \times (332 \\ 20998 \\ \hline 13 \end{array}$$

Similiter, Diviso 6321 per 19, Quotiens 332, ostendit tot Cyclos Indictionum integros absolutos; & Residuus 6, indicat sequentis annum.

$$\begin{array}{r} 332(6 \\ 6321 \times (421 \\ 26908 \\ \hline 6 \end{array}$$

Annus itaque Periodi Julianæ 6321, est Cycli Solaris, 21; Lunaris, 13; & Indictionum, 6.

Exposito verò, Quotus sit annus aliquis in singulis Periodis Solari, Lunari, & Indictionum; Quotus sit ille annus Periodi Julianæ, investigare; (quod imperat hoc Problema:) non ita facile est. Sic tamen fiet.

Sit (verbi gratiâ) Annus 22, in Cyclo Solari; 14, in Lunari; 7, Indictionum.

I. Propter annum 22 Cycli Solaris; manifestum est annum expositum, esse vel annum 22 primi Cycli (adeoque & 22 Periodi Julianæ) vel alterius alicujus post unum pluresve integros absolutos: Adeoque (propter integram Periodum annorum 28) in serie Arithmetica proportionali ab 22, continuo augmento 28, progrediente. Nempe, 22. 50. 78. 106. 134. 162. 190. 218. 246. 274. 302. 330. 358. 386. 414. 442. 470, &c.

II. Propter annum 14 Cycli Lunaris, (cujus tota Periodus est 19;) pariter manifestum est, eundem annum esse in serie Arithmetica ab 14, continuo augmento 19, progrediente; Nempe, 14. 33. 52. 71. 90, &c. Adeoque hujus eo aliquo anno qui sit cum serie priori communis.

III. Propter seriei primæ numeros omnes pares; manifestum est, nonnisi alternos quosque numeros secundæ seriei usui esse posse; nempe 14, 52, 90, &c. Quæ itaque est series ab 14, continuo augmento 38 = 19 \* 2, progrediens. Puta 14. 52. 90. 128. 166. 204. 242. 280. 318. 356. 394. 432. 470, &c.

IV. Cum Numerus 470 sit utrique seriei primæ & tertiæ, communis; Adeoque manifestum sit, eum annum Periodi Julianæ esse, tum 22 Cycli Solaris, tum 14 Lunaris; Nec illud (propter 28 & 19 numeros inter se primos) iterum recurat

nisi

nisi post annos  $532 = 28 * 19$ , continuè recurrentes: Manifestum est, quæsitum annum esse in serie Arithmetica quærendum, ab 470, continuo augmento 532, progrediente. Puta 470, 1002, 1534, 2066, 2598, 3130. 3662, &c.

V. Propter Annum Indictionis 7, (cujus tota Periodus est 15;) Quærendus est idem Numerus in serie ab 7, continuo augmento 15, progrediente. Nempe 7, 22, 37, 52, &c. Adeoque in hujus aliquo numero qui sit seriei quartæ communis.

VI. Cum vero, propter seriem quartam ex solis numeris paribus, nonnisi alterni numeri in serie quinta usui esse possint; puta 22, 52, &c. Quæ est series ab 22, continuo augmento 30 =  $15 * 2$ , progrediens: Erit in hac serie numerus quæsitus. Puta 22, 52, 82, 112, &c. Adeoque hujus numerus aliquis, qui seriei quartæ communis est.

VII. Cum seriei sextæ numeri omnes binario terminentur; constat quartæ numeros illos solos esse utiles, qui binario item terminantur: Qui, cum nonnisi quinto quolibet ab 1002 loco recurrant, incident in seriem ab 1002, continuo augmento  $2660 = 532 * 5$ , progredientem. Nempe 1002, 3662, 6322, &c.

VIII. Examinatis igitur sigillatim seriei septimæ numeris, an etiam seriei sextæ convenient; hoc est, an per 30 divisi relinquant 22; invenietur numero 6322 hoc convenire. Qui itaque cum seriebus omnibus communis sit, Numerus est quæsitus Periodi Julianæ. Quod erat propositum.

Compendio fieri potest hæc ultima examinatio; si, sumptis indiscriminatim (neglecto ordine) notis numerorum, investigetur num, abjectis 3; quoties fieri potest, superfit 1. (id utique fieri debet, propter 30 per 3 divisibilem, &  $22 = 3 * 7, + 1$ .) Quod in 1002 non contingit, (nam  $1 + 2 = 3$ ;) nec in 3662 (nam abjectis 3, 6, 6, restant 2:) sed in 6322 id evenit; nam, abjectis 6, 3, restant  $2 + 2 = 3 + 1$ .

Idem sumptis alio ordine tribus periodis efficitur: Puta, Solari, Indictionum, & Lunari. Nempe,

I. Propter Cychi Solaris annum 22; erit in serie 22, 50, 78, &c.

II. Propter Indictionum annum 7; erit in serie 7, 22, 37, &c.

III. Propter 22 in utraque serie repertum; erit in serie hinc continuanda, continuo augmento,  $420 = 28 * 15$ . Puta 22,

442,

442, 862, 1282, 1702, 2122, 2542, 2962, 3382, 3802, 4222, 4642, 5062, 5482, 5902, 6322, &c.

IV. Propter Cycli Lunaræ annum 14; idem erit in serie ab 14, continuo augmento 19, progrediente. Puta 14, 33, 52, 71, 90, 109, 128, 147, 166, 185, 204, 223, 242, &c.

V. Cum vero, propter seriem tertiam, binario debeat terminari; qui nonnisi decimo quoque loco ab 52 recurrit: erit idem in serie ab 52 continuo augmento  $19 = 19 \times 10$  progrediente. Nempe 52, 242, 432, 622, 812, 1002, 1192, 1382, 1572, 1762, 1952, 2142, 2332, 2522, 2712, 2902, 3092, 3282, 3472, 3662, 3852, 4042, 4232, 4422, 4612, 4802, 4992, 5182, 5372, 5562, 5752, 5942, 6132, 6322, &c.

Adeoquæ 6322, huic seriei cum reliquis communis, est Periodi Julianæ Annus quæsitus.

Similiter, sumptis hoc Ordine Cyclis; Lunari, Indictionum, Solari.

I. Propter Cycli Lunaræ annum 14; erit in serie 14, 33, 52, &c.

II. Propter annum Indictionum 7; in serie 7, 22, 37, 52, &c.

III. Propter 52 in utraque serie repertum; erit in serie ab 52 continuo augmento  $285 = 15 \times 19$ , progrediente. Nempe 52, 337, 622, 907, 1192, &c.

IV. Propter Cycli Solaris annum 22; erit in serie 22, 50, 78, 106, 134, 162, &c.

V. Propter seriem quartam ex solis paribus; in tertia soli utiles erunt alterni numeri; adeoque quæ ab 52, augmento continuo  $570 = 285 \times 2$  progreditur series. Nempe 52, 622, 1192, 1762, 2332, 2902, 3472, 4042, 4612, 5182, 5752, 6322, &c.

VI. Propter seriem quintam binario ubique terminatam; sumendus erit in quarta nonnisi ab 22 quintus quisque continuus numerus, adeoque continuè addendi  $140 = 28 \times 5$ . Nempe 22, 162, 302, 442, 582, 722, 862, 1002, 1142, 1282, 1422, 1562, 1702, 1842, 1982, 2122, 2262, 2402, 2542, 2682, 2822, 2962, 3102, 3242, 3382, 3522, 3662, 3802, 3942, 4082, 4222, 4362, 4502, 4642, 4782, 4922, 5062, 5202, 5342, 5482, 5622, 5762, 5902, 6042, 6182, 6322, &c.

Adeoquæ

Adeoque 6322, cum in singulis seriebus occurrat, est Periodi Julianæ Annus quæsitus.

Et similiter, non magna mutatione procedendum erit quocunque ordine sumantur tres illæ Periodi; puta, Lunari, Solari, Indictionum: vel, Indictionum, Solari, Lunari: vel denique Indictionum, Lunari, Solari. Et pariter, mutatis mutandis, quotusquisque Annus in singulis Periodis exponatur.

Idem compendiosius.

Ne autem necesse sit series istiusmodi in grandioribus numeris conficere; id compendiosius sic fieri possit.

I. Propter Cycli Solaris numerum 28, cujus Periodus est 28; intelligenda erit series, ab 22 inchoata: Nempe 22, 50, 78, &c. continuè addendo 28, hoc est  $19 + 9$ .

II. Vel (hujus loco) 3, 12, 2, 11, 1, 10, (0, vel) 19, 9, 18, 8, 17, 7, 16, 6, 15, 5, 14, &c. Nempe (neglectis semper 19) additis continuè 9, & quoties summa ultra 19 (integram Periodum Lunarem) excurrat, abijciendo 19. (Ut habeatur Annus Cycli Lunaris currentis, qui currentis cuiusque Cycli Solaris anno 22 respondeat.) Estque eousque continuanda hæc series, donec expositus Cycli Lunaris Numerus, puta 14, occurrat. (Non posse autem ultra locos 19 excurrere quin in se redibit; manifestum est.)

III. Cum expositus Cycli Lunaris Numerus 14, seriei secundæ loco 17<sup>o</sup> occurrat; id indicio est, anno 22, Cycli Solaris 17 currentis, annum 14, Cycli Lunaris, convenire: Hoc est, Anno Periodi Julianæ  $470 = 448 (= 28 \times 16) + 22$ . Cumque hoc non iterum recurat (propter 28, & 19, Numeros inter se primos) nisi post annos exinde elapsos  $532 = 28 \times 19$ , & sic deinceps; (quæ est Periodus Dionysiana, ex Solari & Lunari composita:) Intelligenda erit series, ab 470 (hoc est 22 + 448, *hæc*) continuo augmento 532 (hoc est 19 + 513, *hæc*) progrediens; Nempe 470, 1002, 1534, &c.

IV. Vel (hujus loco) 5, 12, 4, 11, 3, 10, 2, 9, 1, 8, (0, vel) 15, 7, &c. Nempe (neglectis Numeri 15 multiplicibus) à Numero 5 ordiendo, & continuè addendo 7; & quoties summa ultra 15 (integram Indictionum Periodum) excurrat, abijciendo 15. Estque eousque continuanda hæc series, donec occurrat

occurrat expositus Cycli Indictionum numerus, puta 7. (Non posse autem ultra loca 15 excurrere, quin in se redibit; constat.)

V. Cumque in serie quarta, expositus Cycli Indictionum numerus 7, loco 12<sup>o</sup> occurrat, adeoque seriei tertiæ eidem loco respondeat; hoc est, numero  $6322 = 5852: (= 532 \times 11) + 470$ : Indicium est, Periodi Julianæ annum 6322, non modo Cycli Solaris esse annum 22, & Lunaris 14, (quod omnibus seriei tertiæ commune est,) sed & Indictionis 7. Quod investigatu erat propositum.

Idem similiter fiet, quocunque ordine sumantur Cycli, puta, Lunari, Solari, Indictionum.

I. Propter Cycli Lunaris annum 14, (cujus tota Periodus est 19;) intelligenda est Periodus ab 14 inchoata; puta 14, 33, 52, &c. addendo continuè 19.

II. Vel (hujus loco) 14, 5, 24, 15, 6, 25, 16, 7, 26, 17, 8, 27, 18, 9, (0, vel) 28, 19, 10, 1, 20, 11, 2, 21, 12, 3, 22, &c. Nempe, quoties summa excrevit ultra 28 (Cyclum Solarem integrum) abiectis 28. Estque hæc series (quæ ultra 28 locos excurrere non potest, quin in se recurrit,) eousque saltim continuanda, donec occurrat expositus Cycli Solaris numerus 22.

III. Cumque hic numerus occurrat, in serie secunda, loco 25<sup>o</sup>: eidem respondet, in serie prima, numerus 470 = 456 (= 19  $\times$  24) + 14. Et procedendum ut in ordine maxime præcedente.

Vel sumantur hoc ordine; Solari, Indictionum, Lunari.

I. Propter Cycli Solaris numerum 22. Intelligatur (ut prius) series, 22, 50, 78, &c. Additis continuè 28 = 15 + 13.

II. Vel (hujus loco) 7, 5, 3, 1, 14, 12, &c. Nempe (neglectis semper 15) additis continuè 13; et, quoties summa ultra 15 (periodum Indictionum integram) excrevit, abjiciendo 15. Estque eousque continuanda series (quæ ultra 15 loca non potest excurrere) donec expositus Cycli Indictionum numerus occurrat; puta 7.

III. Cumque hic jam primo statim loco conspiciatur; indicium est, quod qui huic respondet in prima serie numerus 22, annum designat qui sit tum 22 in Cyclo Solari, tum 7 in Cyclo Indictionum.

num. Cumque hoc non iterum recurat nisi post annos  $420 = 28 \times 15$ : Intelligenda est series 22, 442, 862, &c. additis continuè 420, hoc est  $19 \times 22, + 2$ .

IV. Vel (hujus loco) 3, 5, 7, 9, 11, 13, 15, 17, (0, vel) 19, 2, 4, 6, 8, 10, 12, 14, &c. Nempe (neglectis 19, ejusque multiplicibus) inchoando à 3 (propter  $22 = 19 + 3$ ) additis continuè 2; et, quoties summa excurrit ultra 19 (Cyclum Lunarem integrum) abjiciendo 19. Atque hoc eousque donec occurrat expositus Cycli Lunaris numerus; puta 14. (Nec potest hæc series ultra loca 19 excurrere.)

V. Cumque illic occurrat 14, loco 16°; cui respondet in serie tertia numerus  $632 = 6300 (= 420 \times 15) + 22$ : Erit 6322 Periodi Julianæ annus quæsitus.

Similiter, sumpris hoc ordine Cyclis; Indictionum, Solari, Lunari.

I. Propter Cycli Indictionum annum 7; Intelligatur series 7, 22, 37, 52, &c. additis continuè 15.

II. Vel (hujus loco) 7, 22, 9, 24, 11, 26, &c. Nempe, rejectis, ex summa, 28, quoties hunc numerum excedit: Idque eousque saltem donec expositus Cycli Solaris numerus, puta 22, occurrat. Qui cum hic occurrat loco secundo; cui respondet in serie prima item 22; procedendum ut in exemplo proximè præcedente.

Item, sumpris hoc ordine Cyclis; Lunari, Indictionum, Solari.

I. Propter Cycli Lunaris numerum 14; Intelligatur series 14, 33, 52, &c. Additis continuè 19; hoc est,  $15 + 4$ .

II. Vel (hujus loco) 14, 3, 7, 11, (0, vel) 15, 4, &c. Nempe (neglectis 15) continuè additis 4; &c. ubi summa excrevit ultra 15 (Cyclum Indictionum integrum) abjiciendo 15: Eousque saltem donec occurrat expositus Cycli Indictionum numerus; puta 7.

III. Cumque hic numerus occurrat loco tertio; cui in serie prima respondet 52: Intelligenda series 52, 337, 622, &c. additis continuè  $285 = 19 \times 15$ : hoc est  $28 \times 10, + 5$ .

IV. Vel (hujus loco) 24, 1, 6, 11, 16, 21, 26, 3, 8, 13, 18, 23, (0, vel) 28, 5, 10, 15, 20, 25, 2, 7, 12, 17, 22, &c. Nempe (neglectis 28, ejusque multiplis) incho-



inchoando à 24 (propter  $52 = 28 + 24$ ) additis continuè 9, & quoties summa excreſcit ultra 28 (integrum Cyclum Solarem) abjiciendo 28. Idque eoſque donec occurrat expoſitus Cycli Solaris numerus; puta 22.

V. Cumque hic numerus 22, loco 23<sup>o</sup> conſpiciatur; cui reſpondet in ſerie tertia, numerus 6322 = 6270 (=  $285 \times 22$ ) - 52: Erit hic, numerus in Periodo Juliana quaſitus.

Denique, Sumptis hoc ordine Cyclis; Indiſtionum; Lunari, Solari.

I. Propter Cycli Indiſtionum numerum 7: Intelligenda eſt ſeries 7, 22, 37, 52, &c. additis continuè 15.

II. Vel (huius loco) 7, 3, 18, 14, 10, 6, &c. Nempe, quoties ſumma excedit 19 (Cyclum Lunarem) abjiciendo 19. Atque hoc eoſque donec occurrat expoſitus Cycli Lunaris numerus; puta 14. Qui cum hic conſpiciatur loco quarto, cui in prima ſerie reſpondet 52: procedendum ut in exemplo proximè procedente.

Eadem methodo procedendum, ſi, ultra hos tres Cyclos, etiam quartus, quintus, plureſve accederent; & quaereretur Annus Periodi ex omnibus compoſita.

Hinc elicitur (inter alia) hæc brevis Regula.

I. Expoſito numero Cycli Solaris (vel illius exceſſui ſupra 19, ſi hunc numerum excedat,) addantur continuè 9, (abjeſtis ſemper 19, quoties ultra hunc numerum excurritur,) donec occurrat expoſitus Cycli Lunaris numerus. Sic habetur ſeries numerorum in Cyclo Lunari, expoſito Cycli Solaris numero (in ſucceſſivis periodis recurrenti) reſpondentium.

II. Per numerum locorum huius ſeries, uno minus, multiplicentur 28, & facto addatur expoſitus Cycli Solaris numerus. Sic habetur numerus in Periodo Dionyſiana qualibet, utriſque ſimul Cyclorum Solaris & Lunaris numeris expoſitis reſpondens.

III. Ex invento illo Cycli Dionyſiani numero, abjiciantur quoties fieri poteſt 15, (vel dividatur numerus ille per 15;) & Reſiduo (ſive Subductionis ſive Divisionis,) vel ipſi 15 ſi reſiduum ſit 0, addantur continuè 7, (abjeſtis ſemper 15 quoties ultra hunc numerum excurritur,) donec occurrat expoſitus Cycli Indiſtionum numerus. Sic habetur ſeries numerorum in Cyclo Indiſtionum, invento Cycli Dionyſiani numero (in ſucceſſivis Periodis recurrenti) reſpondentium.

IV. Per



IV. Per numerum locorum hujus seriei, uno minus, multiplicentur 532; & facto addatur inventus Cycli Dionysiani numerus. Sic habetur Periodi Julianæ numerus quæsitus.

Operatio subjicitur.

Cycl. ②. 22 . . . . . 22

Cycl. D. 3, 12, 2, 11, 1, 10, 19, 9, 18, 8, 17, 7, 16, 6, 15, 5, 14

Cycl. Dionys. 470

Cycl. Indict. 5, &c.

22 . . . . . 22.

14 . . . . . 14.

Cycl. Dionys. 470 . . . . . 470.

Cycl. Indict. 5, 12, 4, 11, 3, 10, 2, 9, 1, 8, 15, 7.

Period. Jul. 6322.

28

× 16

168

28

448

+ 22

470

2(5  
470(31  
258  
2

28 × 19 = 532

× 11

532

532

5852

+ 470

6322

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FINIS.

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## ERRATA;

*Qua propter Authoris à Prelo distantiam (quo minus licuerit Chartas, antequam eas absolverant opera, inspicere) admissa sunt; sic emendat Lector.*

**P**Ag. I. lin. II. lege *investigandū*; (ab ipso item alias, ni fallor, solutum.)  
 p. 3. l. 14. libeat. l. 30. dele inter. p. 5. l. 14. se determinetur. l. 26.  
 ponatur. l. 33. revera in ejus. p. 6. l. 30. Δ. Puta. p. 7. l. 7. puncto f. l. 8.  
 occurrens in γ. l. 20. erit. l. 22. dele prius in. l. 28. ipsi Bf. l. 32. ipsam Bf. l. 34. B F  
 ad B f. l. 37. necne. Quamquam. p. 8. l. 2. minor sit quam Δ Γ. l. 11. menta.  
 ibid. proportionalia; sic. l. 18, 19, & 20. pro D N, lege D n. l. 20. muta-  
 tis signis. l. 21. B D, B Δ proportionalē. l. 26. e δ (= Δ B. l. 29. Δ δ  
 — Be. Δ δ — B e :: Δ δ. Δ δ. l. ult. Γ γ — Be. Γ γ — Be :: p. 16.  
 l. ult. Nam ut 2684<sup>7</sup>/<sub>9</sub>. p. 21. l. 31. nono; ut poss. p. 22. l. 18. quarta <sup>2</sup>/<sub>8</sub>;  
 l. antepen. marg. pro 1182, lege 1183. p. 23. l. 6. pro 3616, lege 3613.  
 l. 9. procedendum esset. l. 18. ) Secunda. p. 24. marg. l. 2. pro 2792, lege  
 2793. p. 25. l. 1. suppetat. l. 6. dele <sup>1</sup>/<sub>10</sub>. p. 26. l. 13. lege 25 — 78.  
 p. 27. marg. l. 13. lege 0056579. p. 30. l. antepen. pro 64606, lege 164606.  
 p. 31. l. 10. pro 9882, lege 9082. l. 21. pro 617, lege 657. l. 30. pro 46882.  
 lege 46682. p. 32. l. 15. pro 2891, lege 2861. pag. 35. l. 4. obvium. l. 30.  
 pro 3122, lege 7122. l. 33. haberem. l. ult. pro ut, lege est. p. 36. l. 24.  
 respectivis. p. 37. l. 6. idem atque. marg. l. 11. pro 14917, lege 74917.  
 p. 38. 7. Cumque hoc. marg. l. 7. pro 229, lege 299. p. 39. l. 4. Deno-  
 minatoris, 303. l. antepen. pro 334, lege 224. p. 42. l. 26. evadat. p. 43.  
 l. 7. pro 262, lege 265. ibid. pro 538, lege 238. ibid. pro 289<sup>1</sup>/<sub>3</sub>, lege  
 289 — l. 8. pro 262, lege 265. p. 44. l. 15. pro 17378, lege 17373.  
 lin. 16. pro 17723, lege 17728. l. 28. pro 9258, lege 9208. p. 47. l. 30.  
 pro 33989, lege 32989. p. 49. l. 19. pro 54671, lege 54671. p. 52.  
 (sed quæ signatur 50) l. 27. pro 26109, lege 26609. p. 53. (sed quæ sig-  
 natur 51) l. 21. pro 51698, lege 55698. p. 56. (sed quæ signatur 54)  
 l. 15. seu 8, 5.) l. penult. versa Puta. p. 57. l. 6. Ordineque (non se-  
 cundo quidem, sed) Terio. l. 14. reperitur. l. 20. dele utraque. l. penult.  
 Continuum Incrementum. p. 61. l. 26. pro x) 2, lege x (2.